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FLUTTER OF A CONTROL SURFACE IN SINGLE-DEGREE-OF-FREEDOM BASED ON POTENTIAL FLOW INCLUDING THE EFFECT OF COMPRESSIBILITY

A Thesis

Presented to

the Faculty of the Department of

Engineering

University of Virginia

In Partial Fulfillment

of the Requirements for the Degree

Master of Aeronautical Engineering

by

Harry L. Runyan

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TABLE OF CONTENTS

LIST OF FIGURES

FIGURE PAGE

FIGURE PAGE

LIST OF TABLES

CHAPTER I

THE PROBLEM AND HISTORICAL BACKGROUND

I. THE PROBLEM

Introduction to Problem. In the study of aerodynamic phenomena, the addition of the dimension "time" to steady aerodynamics, has enabled the investigator to examine, theoretically, such problems as transient flows, gusts, dynamic response in flight, maneuvers, dynamic stability and flutter. This thesis is concerned with a phase of this broad subject, specifically with a type of flutter of a control surface in potential flow. To orient the reader, there follows a general description of the mechanism of flutter.

Flutter is an oscillatory instability of a lifting surface, such as an aircraft wing or control surface, which may result in distortion or destruction of the wing or control surface. The energy for the maintenance of growth of the oscillations comes only from the aerodynamic action of the fluid passing over the wing or control surface. By examining the growth of lift on a wing associated with a change in angle of attack, a physical picture of the source of energy may be further clarified as follows. Lift is directly affected by the amount of bound circulation present on the wing, the amount of circulation is determined by the condition (termed the Kutta condition) that the flow must leave the sharp trailing edge of the airfoil smoothly, with no infinite velocity. If the angle of attack is rapidly increased, the bound circulation must change in order to maintain the proper trailing edge condition of no infinite velocity.

the control surface allowed to rotate. It is to be expected that for an actual configuration, many degrees-of-freedom will be present. However, a study of the single-degree-of-freedom system may be significant in assisting in an understanding of the coupled type of flutter, and even under special conditions, single-degree-of-freedom system may be of practical concern.

Purpose of the investigation. It was the purpose of this investigation to (1) explore or show the existence of single-degree-of-freedom flutter of a control surface based on potential theory, and (2) to examine and present the effects of various independent parameters such as Mach number, density of fluid, axis of rotation location, structural damping, and aerodynamic balance on this type of oscillatory instability.

Importance of the study. The general field of flutter is becoming of increasing importance because of the tends of modern aeronautical development. For instance, it is becoming evident that certain phases of flutter are closely allied to the dynamic stability of aircraft. This increase in importance is due in part to the increasing speed ranges, higher operating altitudes, and unusual configurations of modern aircraft.

Almost all activity in flutter research has been focused on the coupled type of flutter and there exists an area of the field that has not been fully examined. This area of the field is single-degree-of-freedom flutter and, in particular, single-degree-of-freedom flutter of a control surface.

Recently single-degree-of-freedom-flutter of a control surface has been experienced in high speed flight, and the explanation of the cause has not been entirely clear, although some investigators have been of the opinion that separated flow is involved. The mechanism of instability presented in this paper, however, should provide a framework for a more complete study, even though it is not necessarily a complete explanation of the cause. For example, this study could provide for a logical grouping of parameters that should be investigated.

II. HISTORICAL BACKGROUND

The first developments in non-stationary aerodynamic theory were made by Wagner[1] in 1925 and by Birnbaum[2] in 1924. These investigators were primarily interested in obtaining the expressions for the aerodynamic forces on an accelerating or oscillating lifting surface and they were not concerned with the conditions under which the dynamical system could become unstable.

The first reference in the literature to single-degree-of-freedom flutter of a wing based on potential flow was made by Glauert[3] in 1929, in a paper in which he derived expressions for the oscillating aerodynamic forces for the incompressible, two-dimensional flow case. Although Glauert made no calculations for a wing oscillating in pitch in two-dimensional flow, he states that for locations of the axis of rotation:

> ...further forward than 0.25 of the chord, the damping moment changes sign at very low frequencies and the oscillation

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1 H. Wagner, Uber die Entstehung des dynamischen Auftriebs von Tragflugeln. Zeitschrift fur Angewandte Mathematik und Mechanik, Vol.5 (1925), pp. 17-35

2 W. Birnbaum, Das ebene Problem des schlagenden Flugels, Zeitschrift fur Angewandte Machematik und Mechanik, Vol. 4 (1924) pp. 277-292

3 H. Glauert, Force and Moment on an Oscillating Aerofoil, Aero. Research Committee Reports and Memorandum, 1242 (Ae 397), March 1929, London p. 15

> of the aerofoil will be maintained by the unstable damping moment. On first thought it might be considered that the moment on the aerofoil in an oscillation of very low frequency should be the same as in uniform circular motion, since the angular acceleration is negligibly small. There is, however, the real physical difference between the two motions: in steady circular motion the circulation round the aerofoil is constant and there is no vortex wake, whereas in an oscillation of very low frequency, when the aerofoil is passing through its mean position with increasing angle of incidence, the circulation has been increasing slowly for a long time and there is vortex wake behind the aerofoil which modifies the conditions of flow.

Following Glauert's work, the subject of single-degree-of-freedom flutter lay dormant and little interest was exhibited in the matter until Possio[4], several years later, made similar observations for the case of two-dimensional supersonic flow. Possio made no calculations, but simply examined the damping moment equation and concluded that single-degree-of-freedom flutter of a wing in supersonic flow was possible for certain positions of the axis of rotation and ranges of Mach number. Following this paper, Garrick and Rubinow[5], in 1946, published a report on the flutter of a wing in supersonic two-dimensional flow. Part of this paper was devoted to single-degree-of-freedom pitching oscillation of a wing. An expression for the damping moment in terms of Mach number and position of the axis of rotation was given, as well as a plot of the ranges of Mach number and pitch axis in which single-degree-of-freedom flutter would be likely to occur. Also of significance in this report by Garrick and Rubinow

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4 C. Possio, L'azione aerodinamica sul profilo oscillante alle velocita ultrasonore. Acta. Pont. Acad. Sci., Vol. I, No. 11, 1937, pp. 93-105

5 I. E. Garrick and S. I. Rubinow, Flutter and Oscillating Air Force Calculations for an Airfoil in a Two-Dimensional Supersonic Flow, Technical Note 1158, NACA, October 1946, pp. 34-36

is their remark that

> ...It may be appropriate to mention that a similar torsional instability is theoretically indicated even in the subsonic (incompressible) case...

This appears to be the first time, since 1929, that the possibility of single-degree-of-freedom flutter in incompressible flow was mentioned.

The preceding investigations dealt with two-dimensional flow. Watkins[6], in 1949, made a study of some three dimensional effects on a rectangular wing oscillating in supersonic flow. He presented plots showing the locations of pitch axis and Mach number for various aspect ratios in which the oscillation could occur. The effect of decreasing the aspect ratio decreased the ranges of Mach number and pitch axis location in which the oscillation could occur, however, decreasing the aspect ratio did not completely eliminate the possibility of single-degree-of-freedom flutter.

The results of a study of the case of a wing oscillating in pitch in two-dimensional, incompressible flow has recently been reported by Smilg[7]. He presented the results of calculations showing the ranges of location of the axis of rotation and of the values of an inertia parameter for which the oscillations are indicated. Smilg states in his conclusions that

> ...The existence of this type of instability would not be predicted by pseudo-static aerodynamic forces now commonly employed in airplane stability analysis.

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6 Charles E. Watkins, Effect of Aspect Ratio on Undamped Torsional Oscillations of a Thin Rectangular Wing in Supersonic Flow, NACA TN 1895, 1949

7 B. Smilg, The Instability of Pitching Oscillations of an Airfoil in Subsonic Incompressible Potential Flow, Journal of Aeronautical Science, November 1949, Vol. 16, No. 11

Following this paper, Runyan[8] extended the calculations of Smilg for a pitching wing to include the effect of Mach number and structural damping. It was shown that Mach number had a very marked effect and yields oscillatory instabilities over a wider range of an inertia parameter. Consequently, a configuration that would be stable in incompressible flow could become unstable at the higher subsonic speeds.

The preceding historical background refers to single-degree-of-freedom flutter of a wing and not to control surfaces. A recent study of the oscillatory instability of a control surface alone, not based on potential flow, was made by Smilg[9], in which he discussed rather broadly some possible causes of control surface oscillations that have been found on some high speed aircraft. He concluded that such factors as flow separation and shock wave location were important factors in governing the oscillations. There apparently do not exist, to the author's knowledge, studies demonstrating the existence of single-degree-of-freedom control surface oscillation based on potential flow, and this subject forms the basis of the present thesis.

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8 Harry L. Runyan, Single-Degree-of-Freedom-Flutter Calculations for a Wing in Subsonic Potential Flow and Comparison with an Experiment, NACA, Unpublished TN, April 1951

9 Benjamin Smilg, The Prevention of Aileron Oscillations at Transonic Speeds, AAF Technical Report 5570, Army Air Forces, Air Materiel Command, Wright Field, Dayton, Ohio

CHAPTER II

INTRODUCTORY CONSIDERATIONS FOR FLUTTER AND EQUATION OF EQUILIBRIUM FOR A CONTROL SURFACE

In this chapter, a general linear single-degree-of-freedom system consisting of a mass, spring and damper is discussed. The system including aerodynamic oscillating air forces with reference to a wing and to a control surface is then discussed. Finally the significance of the equations of motions with regard to flutter is given. Symbols are defined as they occur, but for convenience are also grouped together and presented in the appendix.

I. SINGLE-DEGREE-OF-FREEDOM VIBRATION OF A SPRING, MASS, DASHPOT SYSTEM

The linear differential equation for the equilibrium of forces for a spring, mass, dashpot system of Fig. 1(a) may be determined from Newton's second law $m \frac{dv}{dt} = \sum F$ as

$$m\ddot{x} + d\dot{x} + k'x = 0 \qquad \text{Eq. 1}$$

where m is the mass of the system, x is the displacement from an equilibrium position (the dot signifies differentiation with respect to time), d is the coefficient of damping, and k' is the structural restraint or spring constant. The solution to equation 1 may be put in the form

$$x = e^{-\frac{d}{2m}t}\left(C_1 \cos qt + C_2 \sin qt\right) \qquad \text{Eq. 2}$$

where C_1 and C_2 are constants and may be determined by the boundary condition and $q=\sqrt{\frac{k'}{m}-\frac{d^2}{m^2}}$ (This solution may be found in a standard textbook on differential equations or in a book on vibration of mechanical systems.) The motion of the mass m is then seen to be dependent on the sign of the damping coefficient d. If d is a positive quantity, the motion is damped, that is the amplitude motion of the mass m will decrease with increasing time. However, if d is negative, the amplitude of the motion of the mass will increase with increasing time, and this condition is usually termed an undamped or a negatively damped condition. The situation of d being negative is not usually the physical case, but can occur under special circumstances where sources of energy may be present and such a condition is termed self excited. If d is zero, harmonic oscillations may exist, which correspond to a borderline condition between damped and undamped motion. Therefore, the type of motion of mass m, may be determined by simply examining the sign of the damping coefficient d, without the necessity of determining the complete solution as given by equation 2.

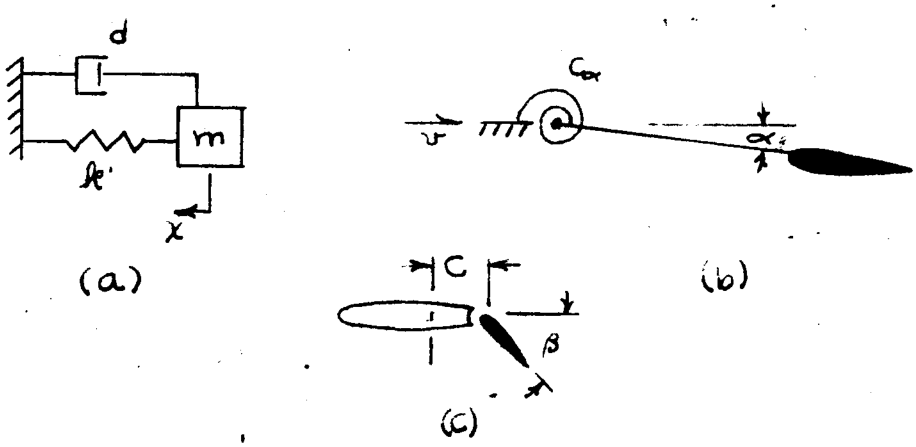

FIGURE 1

SEVERAL TYPES OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS

II. SINGLE-DEGREE-OF-FREEDOM VIBRATION AND FLUTTER OF A WING.

In Fig. 1(b) is shown a diagram representing a single-degree-of-freedom system similar to that in Fig. 1(a) with, however, the mass system replaced by a rigid wing. The wing is considered rigid and can rotate only about the axis of rotation a. The angular rotation of the wing from an equilibrium condition is denoted by α, and the spring constant is denoted by C_α, and the viscous damping moment coefficient is given by D. By application of Newton's second law for rotation about an axis of rotation a, there is obtained for the equilibrium of moments

$$I_\alpha \ddot{\alpha} + D\dot{\alpha} + C_\alpha \alpha = 0 \qquad \text{Eq. 3}$$

where I_α is the mass moment of inertia of the system about the axis of rotation a.

Now, if the wing is in an airstream of velocity v, the fluid passing over the wing creates aerodynamic moments (see Section I, Chapter I) which must be included in equation 3. In a general descriptive form, the equilibrium of moments may be written as

$$(I_\alpha + A_1)\ddot{\alpha} + (D + A_2)\dot{\alpha} + (C_\alpha + A_3)\alpha = 0 \qquad \text{Eq. 4}$$

A_1 is, effectively, an aerodynamic inertia term, A_2 an aerodynamic damping term, and A_3 an aerodynamic stiffness term. These three coefficients are functions of Mach number, location of axis of rotation, and a reduced frequency parameter $k = \frac{b\omega}{v}$, where b is the half chord,

ω the circular frequency and v is the fluid velocity. Ordinarily, the air force coefficients for flutter are not presented in the form of the coefficients A_1, A_2, and A_3, but rather the total aerodynamic moment M_α is presented. Equation 4 may then be written as

$$I_\alpha \ddot{\alpha} + D\dot{\alpha} + C_\alpha \alpha = M_\alpha$$

where

$$M_\alpha = -(A_1 \ddot{\alpha} + A_2 \dot{\alpha} + A_3 \alpha)$$

III. SINGLE-DEGREE-OF-FREEDOM FLUTTER OF A CONTROL SURFACE

In Fig. 1(c) is shown a single-degree-of-freedom control surface system, in which the wing is completely restrained, but the control surface is allowed to rotate about its hinge line c (c is measured from the mid-chord, based on the half-chord, and positive rearward.) The control surface is restrained by a spring, having a stiffness C_β. For an actual control surface, the flexibility of the control cables and attachments would provide the structural stiffness about the hinge line. The mass moment of inertia of the control surface about its hinge line is denoted by I_β, and the angular rotation of the control surface from some equilibrium is given by β. By similar considerations as made for the oscillating wing of section II, the equation for the equilibrium of moments for a control surface in an airstream of velocity v, may be derived by use of Newton's second law to obtain

$$(I_\beta + B_1)\ddot{\beta} + (D + B_2)\dot{\beta} + (C_\beta + B_3) = 0 \qquad \text{Eq. 5}$$

where the aerodynamic coefficients B_1, B_2, and B_3 are functions of Mach number M, control surface hinge location c, and the reduced frequency parameter k. The total aerodynamic moment M is again the sum of the component parts so that

$$I_\beta \ddot{\beta} + D\dot{\beta} + C_\beta \beta = M_\beta \qquad \text{Eq. 6}$$

where

$$M_\beta = -(B_1\ddot{\beta} + B_2\dot{\beta} + B_3\beta)$$

It is a matter of great convenience, in oscillatory non-stationary aerodynamics to employ the complex notation to represent the motion and forces. The use of complex quantities like $e^{i\omega t}$ in alternating electrical current theory has been widely adopted. In utilizing the concept of the complex plane, the moment may be written as a vector, drawn from and rotating about, the origin. This moment vector may be written in terms of its various components along the real and imaginary axis. Accordingly, the phase relation between the moment and the displacement is given by the angle between the moment vector and the displacement vector. As an illustration of this use, equation 6 is plotted in Fig. 2. Equation 6 may be put in a different form by the substitution of $\beta = \beta_0 e^{i\omega t}$ to obtain

$$-\omega^2 I_\beta \beta + iD\omega\beta + C_\beta \beta = M_\beta = (M_{\beta R} + iM_{\beta I})\beta$$

and where M_β has been written in terms of a real and imaginary part.

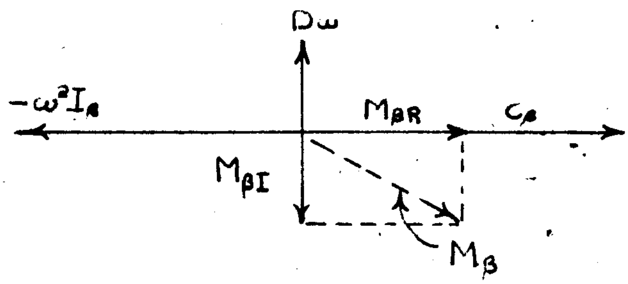

FIGURE 2

VECTORIAL REPRESENTATION OF MOMENTS ON WING

For equilibrium, the summation of the moments along the real axis must be zero, as well as the summation of moments along the imaginary axis. Therefore, two equations may be written as follows

$$\overline{R}_{b\beta} = (-\omega^2 I_\beta + C_\beta - M_{\beta R})\beta = 0 \qquad \text{Eq. 7}$$

and

$$I_{b\beta} = (\omega D - M_{\beta I})\beta = 0 \qquad \text{Eq. 8}$$

The structural damping moment usually found to be of significance in flutter work is not of the viscous type ωD, but rather a type that is not a function of frequency and is a function of the spring moment $C_\beta \beta$. The structural damping moment is then written as $g_\beta C_\beta \beta$, where $g_\beta = \frac{\delta}{\pi}$ and δ

is the logarithmic decrement. Equation 8, would then be

$$I_{b\beta} = (g_\beta c_\beta - M_{\beta I})\beta$$ Eq. 8(a)

Equation 6 can now be written in terms of its components

$$\bar{R}_{b\beta} + i\, I_{b\beta} = 0$$ Eq. 9

The significance of the imaginary component $I_{b\beta}$ may be seen by examining equation 1 and substituting $x = x_0 e^{i\omega t}$ into the equation to give

$$-\omega^2 m x + i d\omega x + k' x = 0$$

It is thus seen that the coefficient of i is the damping coefficient. Similarly, $I_{b\beta}$ represents the damping part of the control surface equation and the vanishing of $I_{b\beta}$ corresponds to a borderline condition between damped and undamped oscillation as was discussed in section I, Chapter II. The real part $\bar{R}_{b\beta}$ is termed the frequency equation from which the frequency of oscillation may be determined.

The procedure for the solution of equation 9 is, therefore, first to determine $I_{b\beta} = 0$ for any set of the parameters, M, c and k. If such a condition is found, indicating the possibility of an instability, the real equation may be solved for the frequency of oscillation and finally for the fluid velocity.

In the following chapters, expressions for $\bar{R}_{b\beta}$ and $I_{b\beta}$ will be explicitly given for a control surface in incompressible flow and in compressible flow.

CHAPTER III

SINGLE DEGREE OF FREEDOM OSCILLATION OF A CONTROL SURFACE IN AN INCOMPRESSIBLE FLUID

The flutter of a control surface in an incompressible fluid is treated in this chapter. The control surface with zero structural damping is first explained, and this is followed by a section in which the effect of structural damping is shown. Finally, some effects of control surface aerodynamic balance are discussed.

I. SINGLE DEGREE OF FREEDOM FLUTTER WITH ZERO STRUCTURAL DAMPING

Equation of equilibrium and method of solution. In Chapter II, it was shown that by the use of the complex notation, the moment equation may be separated into two components so that

$$\left(\bar{R}_{b\beta} + i\,I_{b\beta}\right)\beta = 0$$

Expressions for $\bar{R}_{b\beta}$ and $I_{b\beta}$, without structural damping, were derived by Theodorsen[10] and were later extended to include structural damping by Theodorsen and Garrick[11] and are given as follows

$$\bar{R}_{b\beta} = -\left(\frac{I_\beta}{\pi\rho b^4} - \frac{T_3}{\pi^2}\right) + \frac{1}{k^2\pi^2}\left(T_5 - T_4 T_{10}\right) - \frac{T_{12}}{2\pi}\left(\frac{T_{11}}{2\pi}\frac{2G}{k} - \frac{T_{10}}{\pi}\frac{2F}{k^2}\right) + \frac{I_\beta}{\pi\rho b^4}\left(\frac{\omega}{\omega_\beta}\right)^2$$

Eq. 10

$$I_{b\beta} = -\frac{1}{k}\left[\frac{T_{12}}{2\pi}\left(\frac{T_{10}}{\pi}\frac{2G}{k} - \frac{T_{11}}{2\pi}2F\right) - \frac{1}{2\pi^2}T_4 T_{11}\right] + g_\beta \frac{I_\beta}{\pi\rho b^4}\left(\frac{\omega_\beta}{\omega}\right)^2$$

where I_β is the mass moment of inertia of the control surface about the hinge line c, ρ the fluid density, k ($= \frac{b\omega}{v}$) the reduced frequency,

10 Theodore Theodorsen, General Theory of Aerodynamic Instability and the Mechanism of Flutter, NACA Report No. 496 (Superintendent of Documents, Wash. D.C.)

11 T. Theodorsen and I. E. Garrick, Mechanism of Flutter - A Theoretical and Experimental Investigation of the Flutter Problem, NACA Technical Report 685, 1950 (Superintendent of Documents, Wash. D. C.)

ω_β the natural circular frequency of the control surface about its hinge line, and ω the circular frequency at flutter. The T coefficients are transcendental functions of c and are given in the appendix of the report by Theodorsen and Garrick. The aerodynamic coefficients B_1, B_2, and B_3 of equation 5 could be written in terms of the T functions, so that the T coefficients represent the various aerodynamic inertia, damping, and stiffness terms. The F and G functions appearing in equation 10 are transcendental functions of the reduced frequency k and were first derived by Theodorsen. These quantities are due to the harmonically varying vortex wake which is shed from the trailing edge of the wing. (See Chapter I, Section I) For the stationary case, $k=0$ and $F=\frac{1}{2}$ and $G=0$.

The values $\bar{R}_{b\beta}$ and $I_{b\beta}$ were derived on the basis of linearized aerodynamic theory. The basic equations of aerodynamics are non-linear, that is they involve products or powers of the independent variable. Solution to these non-linear equations have not in general, been obtained and in order to obtain practical results, most investigators have found it necessary to resort to linearization of the equation. The effect of linearization in the present case is to (1) restrict the oscillation amplitude to small values, and (2) reduce the airfoil and control surface thickness to a line. Consequently, the effect of control surface shape is not taken into account.

The non-trivial solution of equation 9 for $g_\beta = 0$ is

$$\bar{R}_{b\beta}=0 \quad \text{and} \quad I_{b\beta}=0$$

since both the real and imaginary parts must both be zero. $I_{b\beta}$ represents an out-of-phase moment on the control surface, positive values of which indicate a damped or stable system, while negative quantities indicate an undamped or driving moment. If $I_{b\beta} = 0$, a borderline condition exists between damped and undamped oscillations; and this is the condition usually sought in flutter analysis. The equation $\overline{R}_{b\beta} = 0$ is the inphase moment from which the frequency of the oscillation and the flutter speed may be determined.

The imaginary part $I_{b\beta} = 0$ is a function of both $1/k$, the reduced velocity, and F and G, which are given by Theodorsen[12]. Since F and G are transcendental functions of $1/k$, a trial process is usually the most convenient method of solving this equation. Assume a value of $1/k$, which determines a set of F and G values, and compute $I_{b\beta}$. Repeat for several values of $1/k$, plot $I_{b\beta}$ against $1/k$, and determine the value of $1/k$ at which $I_{b\beta} = 0$. This is the value of $1/k$ at which the oscillation could occur. The frequency of oscillation may now be determined from the equation $\overline{R}_{b\beta} = 0$, or in another form

$$\left(\frac{\omega}{\omega_\beta}\right)^2 = \frac{1}{1 - M_r \dfrac{\pi \rho b^4}{I_\beta}}$$

where

$$M_r = \frac{T_{12}}{2\pi}\left(\frac{T_{11}}{2\pi}\frac{2G}{k} - \frac{T_{10}}{\pi}\frac{2F}{k^2}\right) - \frac{T_3}{\pi^2} - \frac{1}{k^2\pi^2}\left(T_5 - T_4 T_{10}\right)$$

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12 Theodore Theodorsen, Loc. Cit.

and where the T coefficients are functions only of the control surface hinge location c. These coefficients are tabulated in the Technical Report by Theodorsen[13].

Having determined the frequency of oscillation, the flutter speed parameter $v/b\omega$ is obtained from the equation

$$\frac{v}{b\omega_\beta} = \frac{1}{k}\left(\frac{\omega}{\omega_\beta}\right).$$

where $1/k$ is the value at which $I_{b\beta} = 0$.

Results. Calculations based on the above equations have been made for various locations of the control surface axis of rotation. The results of the computations are presented in Fig. 3 and Fig. 4 for positions of the control surface axis of rotation c = .8, .4, 0, and -1.

In Fig. 3, the flutter speed parameter $v/b\omega_\beta$ is plotted against an inertia parameter $I_\beta/\pi\rho b^4$. The stable region is below or to the left of each curve, and the unstable region is above or to the right. For small values of the inertia parameter to the left of the vertical asymptote, the control surface would be stable at all speeds. If the inertia parameter increases, for instance by an increase in altitude, a value corresponding to the vertical asymptote, at which the flutter speed is infinite, would be reached. In Fig. 3a (c = .8) this value is $I_\beta/\pi\rho b^4 = 7.58$, and it increases to 550 (Fig. 3d) for c = -1, corresponding to a wing oscillating about its leading edge. A slight increase in $I_\beta/\pi\rho b^4$ beyond the vertical asymptote would result in a very rapid decrease in the flutter speed. For values of $I_\beta/\pi\rho b^4$ approaching infinity, $v/b\omega_\beta$ approaches a value equal

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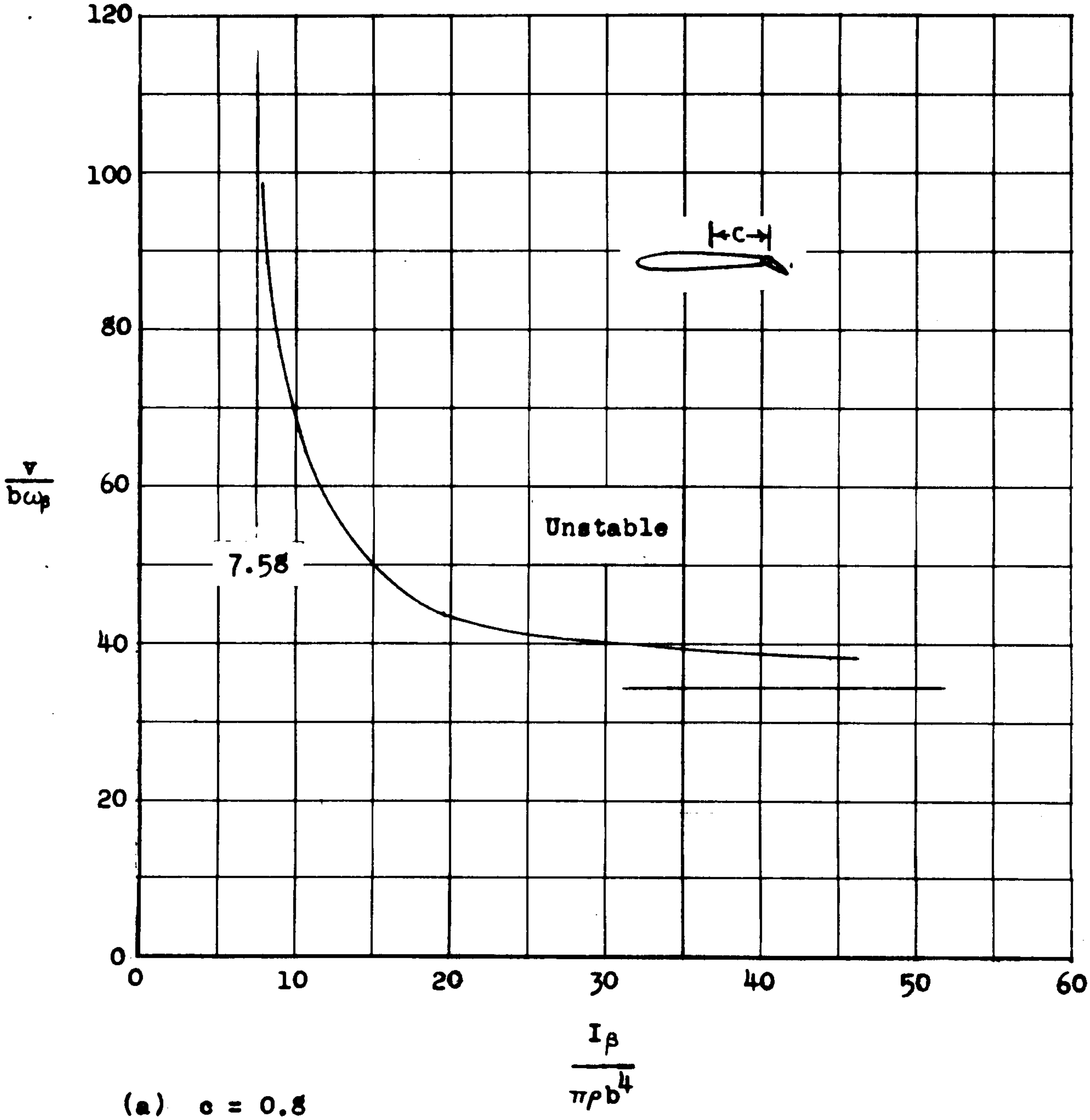
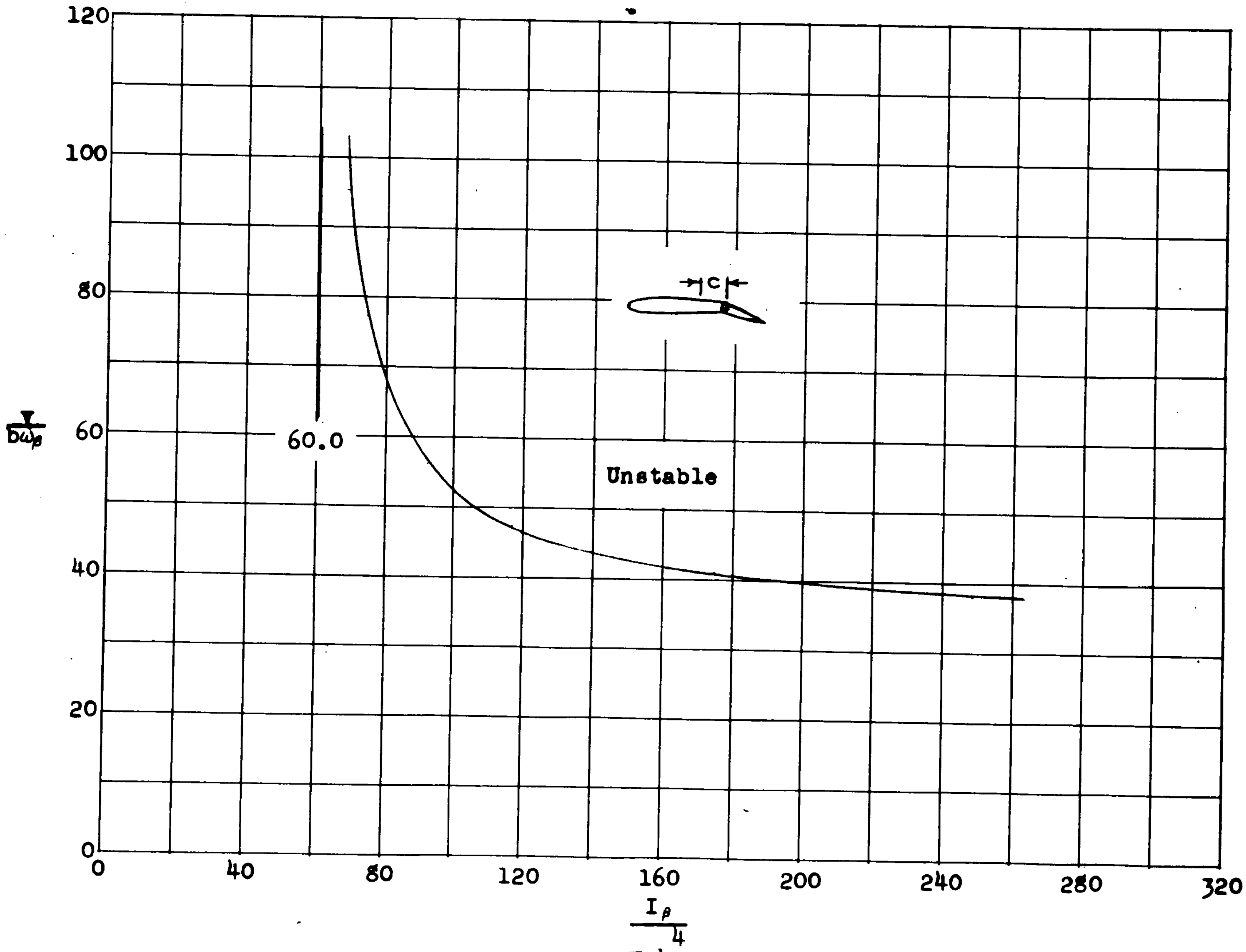
13 Theodore Theodorsen, Loc. Cit.

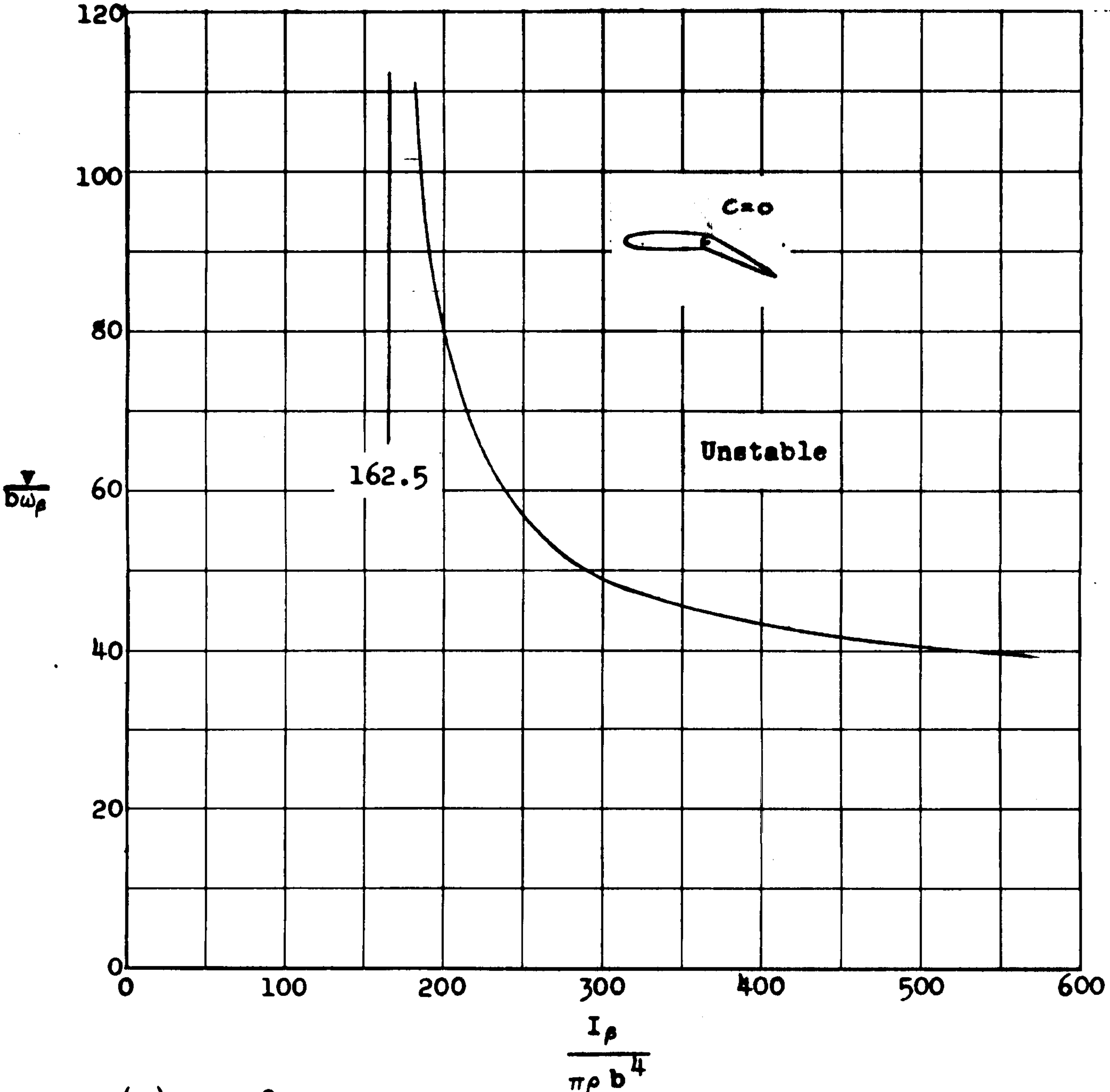


(a) c = 0.8

Figure 3.- Plot of flutter speed parameter $\frac{v}{b\omega_\beta}$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for various axes of rotation of a control surface in incompressible flow.

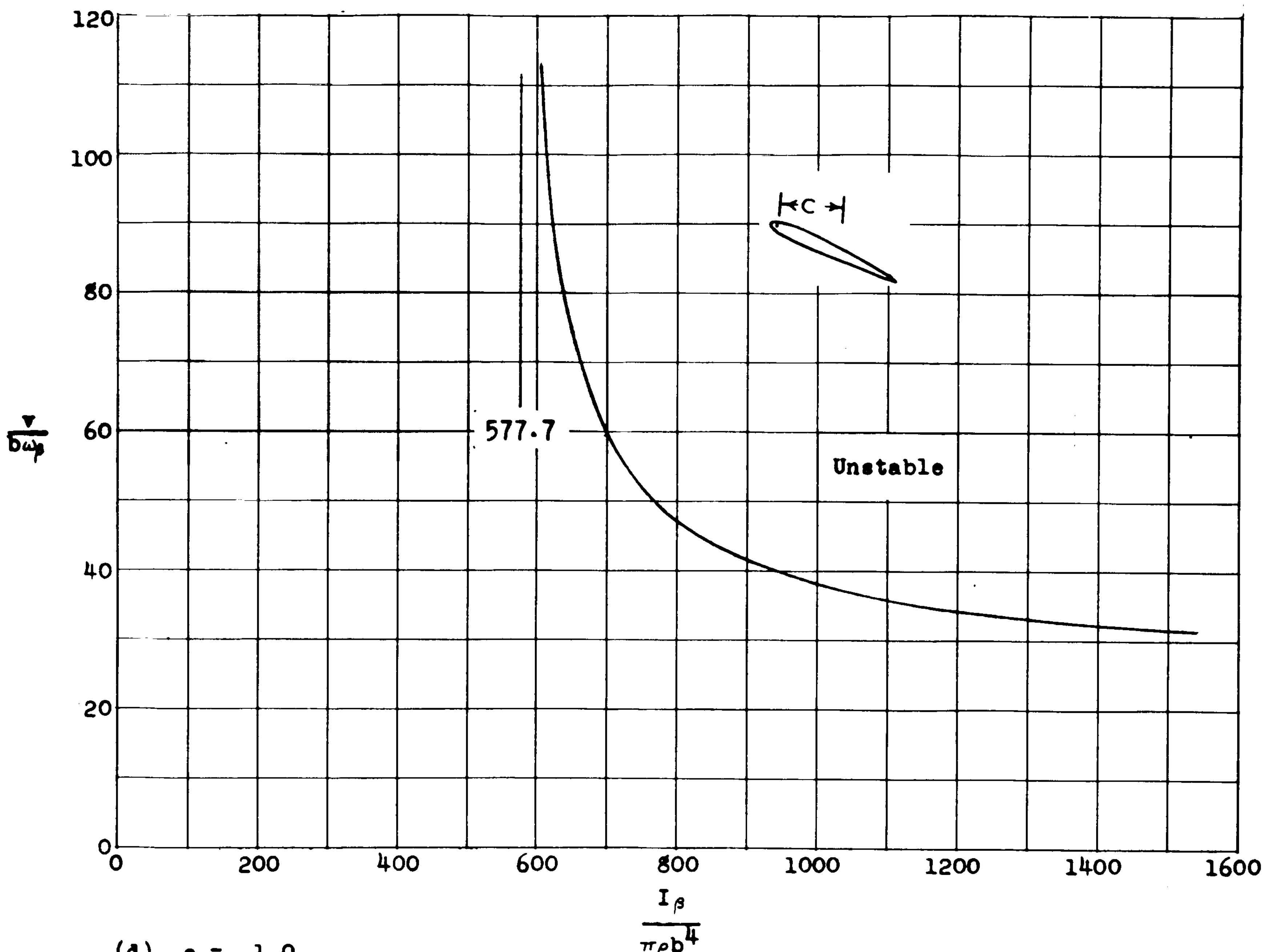

120
100
80
60
40
20
0
0
40
80
120
160
200
240
280
320
$\frac{V}{b\omega_\beta}$
$\frac{I_\beta}{\pi\rho b^4}$
60.0
Unstable
c

(b) c = 0.4



(c) c = 0

Figure 3 - Continued.



(d) c = -1.0

Figure 3.- Concluded.

to $v/b\omega$ at which the oscillation occurs. Consequently, the frequency of oscillation will approach the natural frequency of the system for large values of the inertia parameter. It should be noted that the oscillation occurs at a constant value of $v/b\omega$, consequently, the wave length of the oscillating wake is independent of density (or altitude) change.

The frequency ratio $(\omega/\omega_\beta)^2$ is plotted against the inertia parameter $I_\beta/\pi\rho b^4$ on Fig. 4 for the same axes of rotation as for Fig. 3. The unstable region is above or to the right of a curve. The vertical asymptotes are the same as for Fig. 3, but the horizontal asymptote is 1. This indicates that for very large values of $I_\beta/\pi\rho b^4$, the frequency of oscillation approaches the natural frequency of the control surface.

On Fig. 5, the value of the reduced velocity $1/k$ is plotted against the axis of rotation of the control surface.

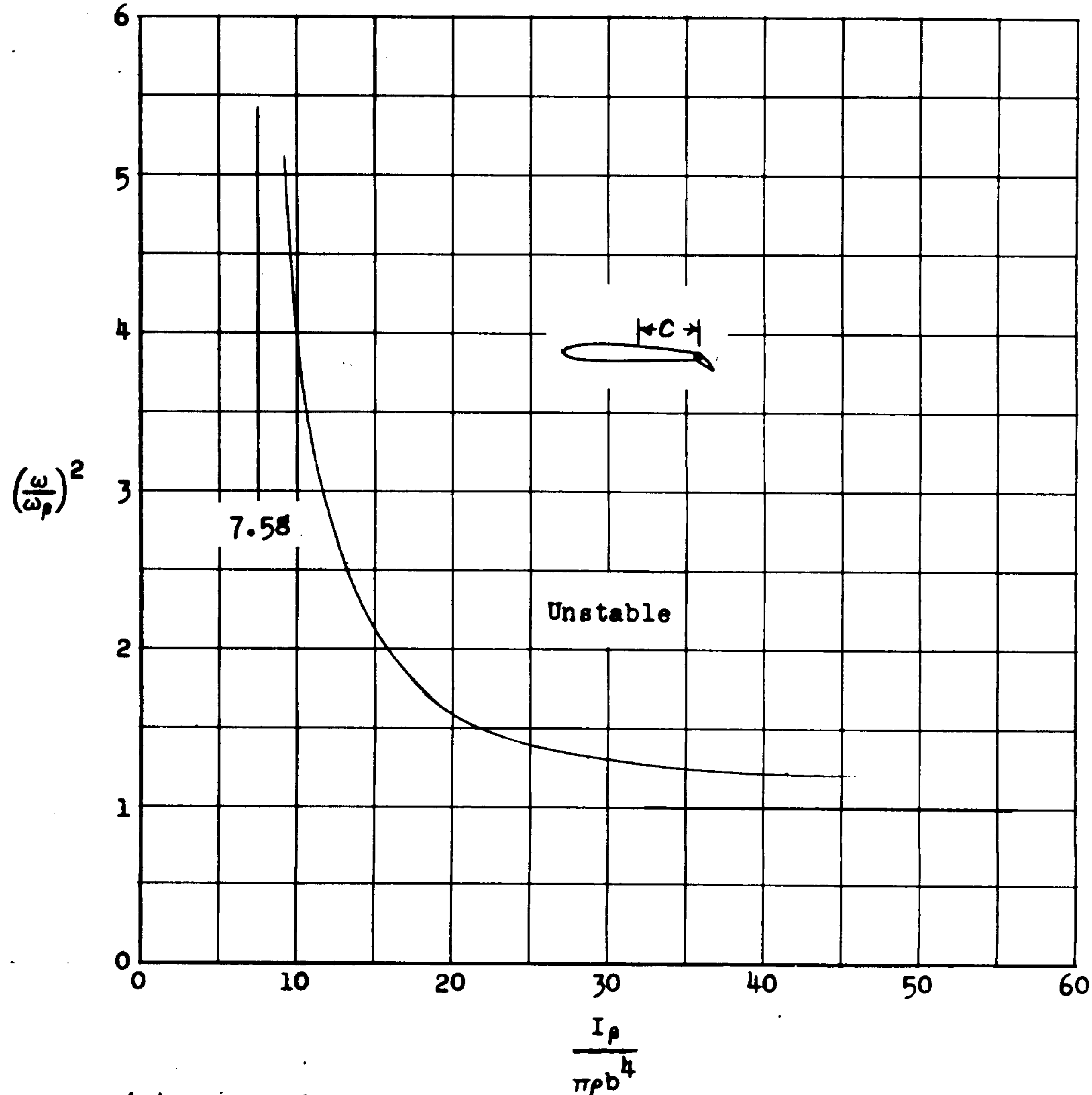
On Fig. 6, the minimum value of the inertia parameter (vertical asymptote) at which the oscillation could occur is plotted against control surface axis of rotation. The value of the inertia parameter increases as the aileron to chord ratio increases. This plot is of particular significance with regard to the zero structural restraint case ($\omega_\beta = 0$). If equation 4 is inverted to obtain

$$\left(\frac{\omega_\beta}{\omega}\right)^2 = 1 - M_r \frac{\pi\rho b^4}{I_\beta}$$

and $\omega_\beta = 0$, there is obtained

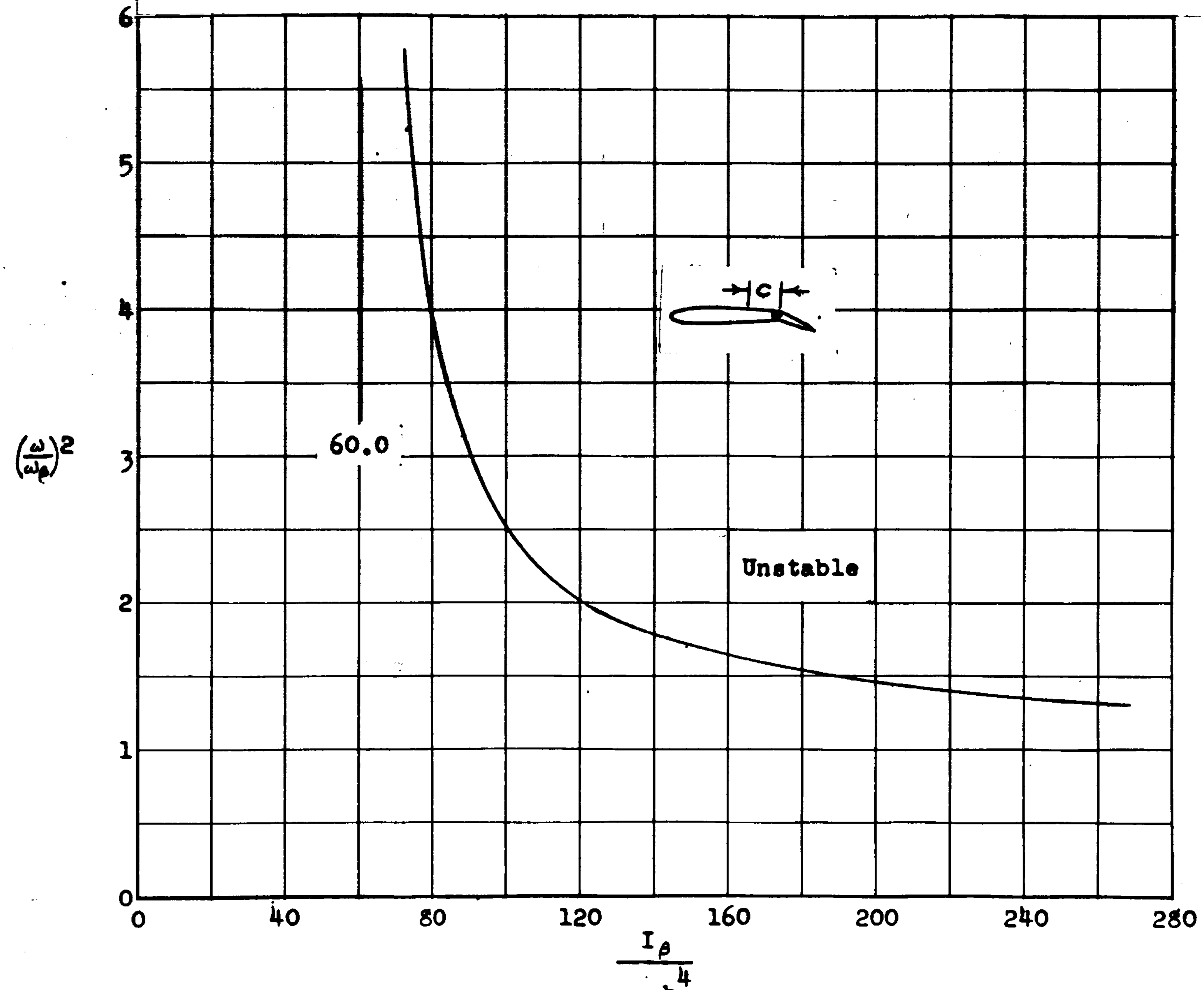
$$M_r = \frac{I_\beta}{\pi\rho b^4}$$

This indicates that if $I_\beta/\pi\rho b^4$ is equal to or greater than M_r an oscillation is possible. The frequency of oscillation is then a direct



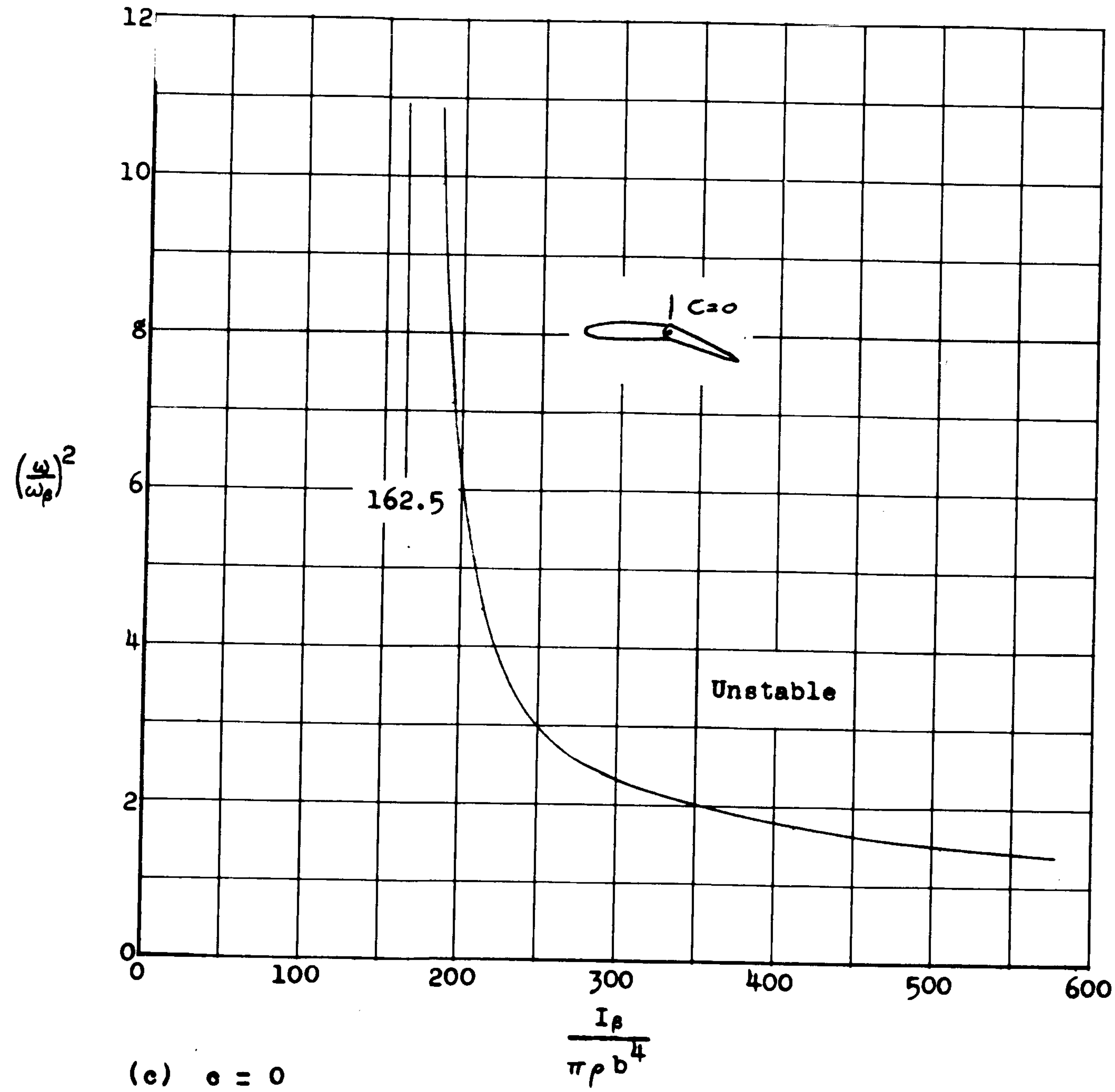
(a) c = 0.8

Figure 4 .- Plot of flutter frequency ratio $\left(\frac{\omega}{\omega_\beta}\right)^2$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for various axes of rotation of a control surface in incompressible flow.



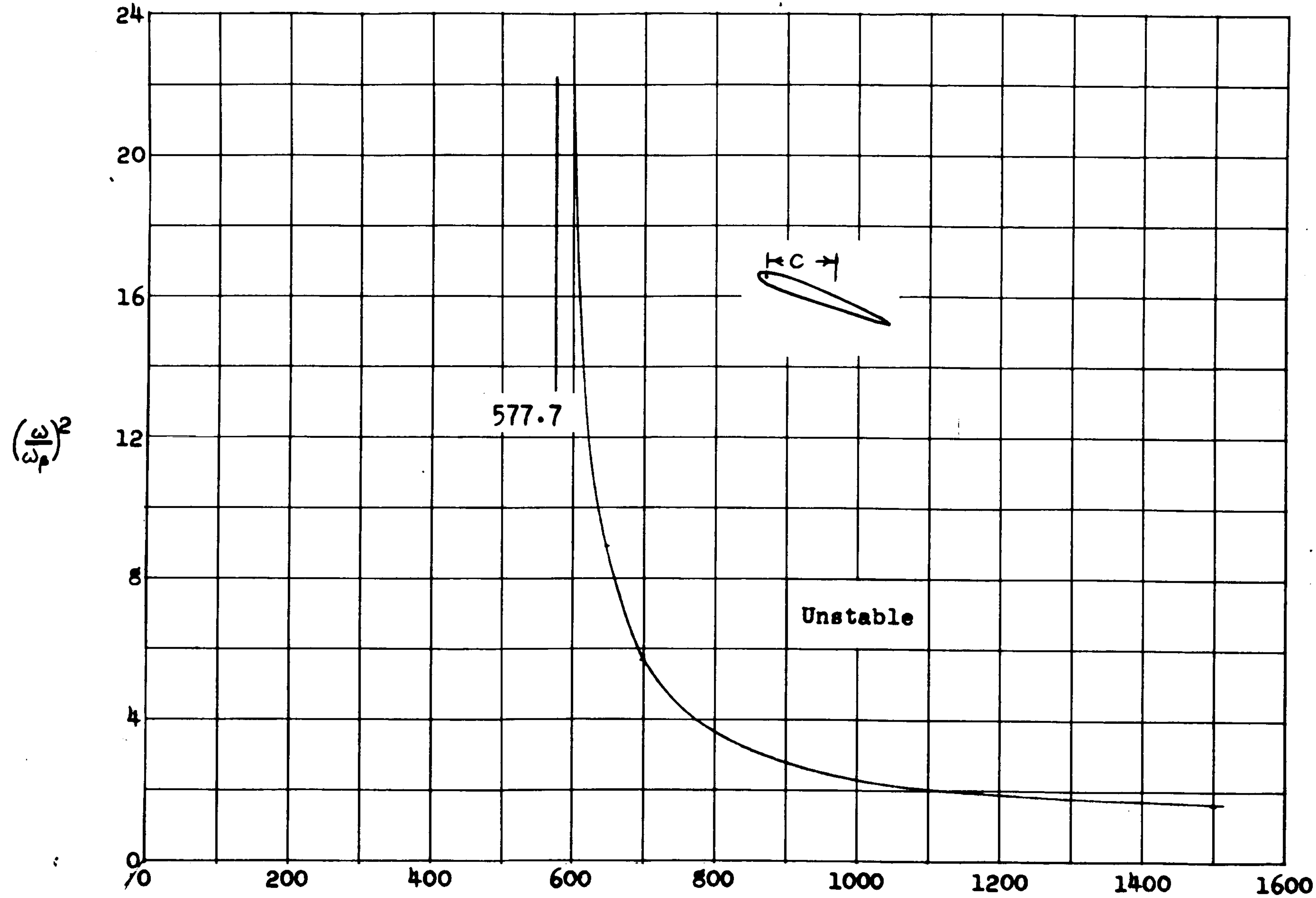
6
5
4
3
2
1
0
$\left(\frac{\omega}{\omega_\beta}\right)^2$
60.0
c
Unstable
0
40
80
120
160
200
240
280
$\frac{I_\beta}{\pi\rho b^4}$

(b) c = 0.4



(c) c = 0

Figure 4.- Continued.



(d) c = -1.0

Figure 4. Concluded.

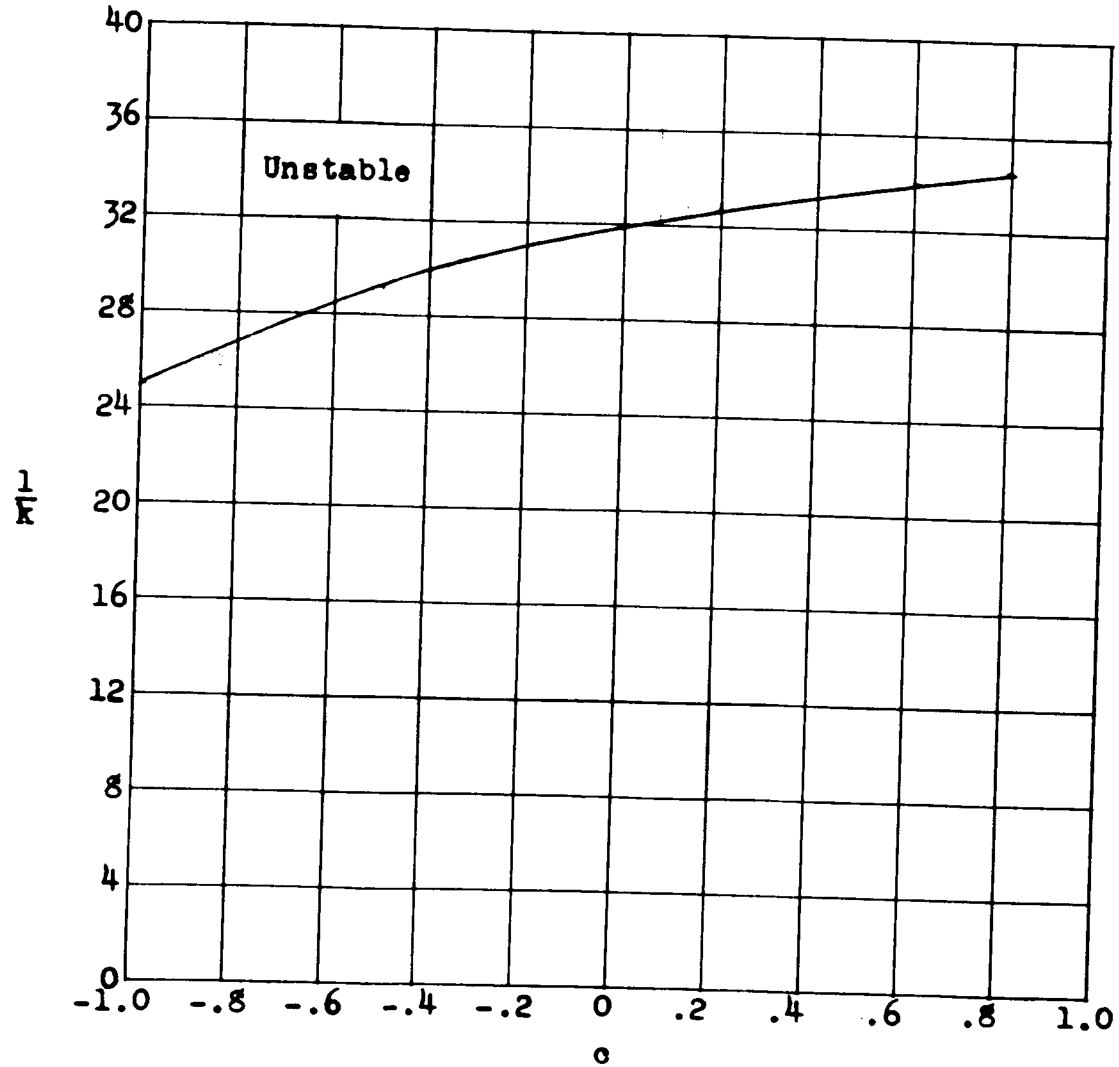


Figure 5 .- Reduced velocity $\frac{1}{k}$ at which oscillation will occur plotted against control surface axes of rotation c for incompressible flow.

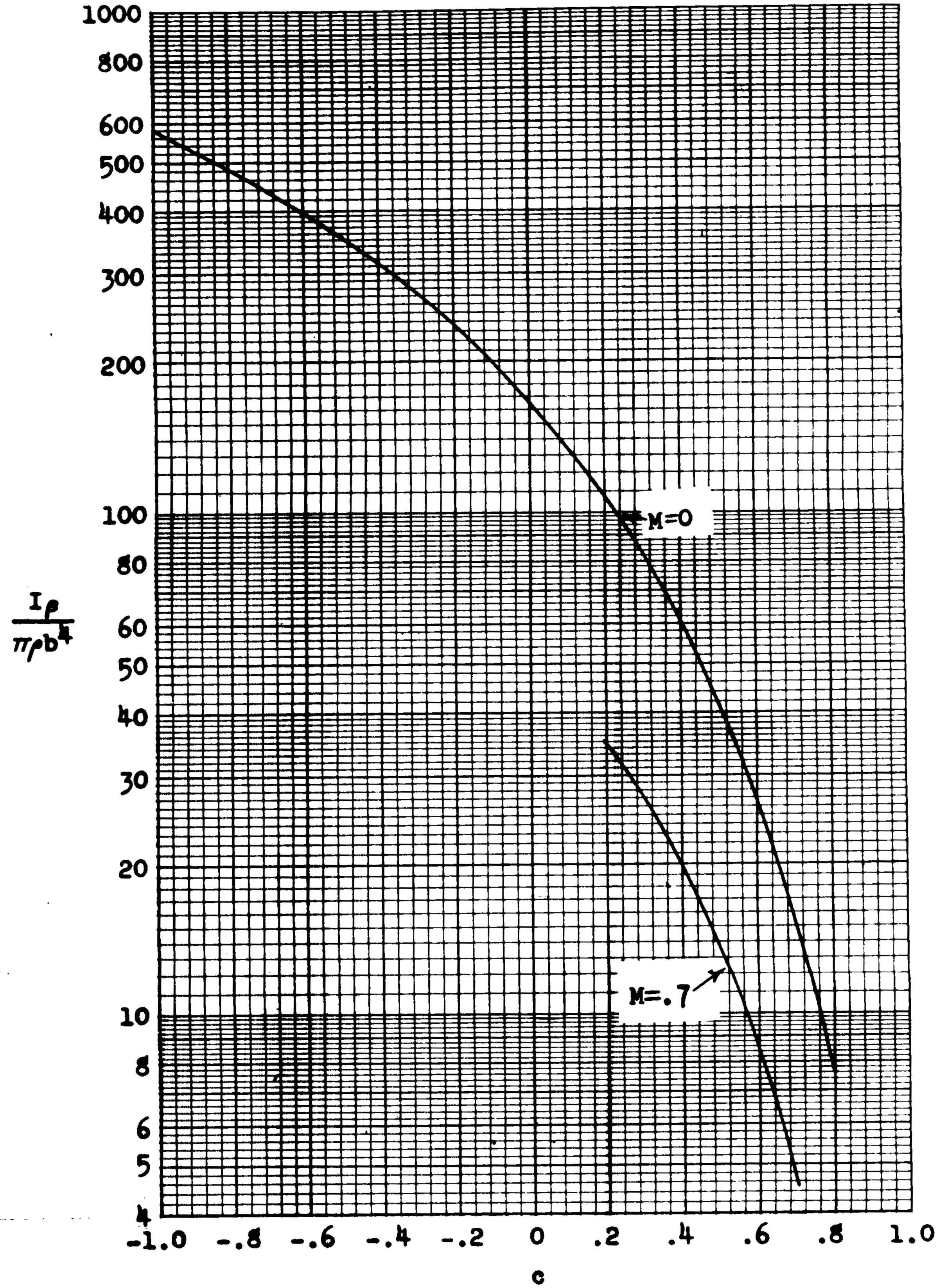


Figure 6.- Plot of asymptotic value of inertia parameter $\frac{I_\beta}{\pi \rho b^4}$ against control surface axis of rotation c for M = 0.

function of the velocity, and flutter can theoretically occur at zero airspeed as hown by the relation

$$v = b\omega/k$$

where $1/k$ is the value that satisfied $I_{b_\beta} = 0$

II. EFFECT OF STRUCTURAL DAMPING

The effect of structural damping, g_β, is considered in this section. From an examination of equation 10, it can be seen that the damping coefficient appears only in the imaginary equation, I_{b_β}. The type of damping assumed is one that is in phase with the velocity $\dot{\beta}$ but proportional to the angular displacement, β.

A convenient method of solving $I_{b_\beta} = 0$ with the damping term included is given in a report by Smilg and Wasserman[14]. The procedure consists of assuming several values of $1/k$ and computing the damping g_β. After several assumptions, the damping g_β is plotted against $1/k$ as shown in Fig. 7 until the value of damping g_β changes sign. Values of $1/k$ may than be determined for a given value of g_β.

Results. The results of computation for one location of the axis of rotation $e = .7$ are presented in Fig. 8 and Fig. 9. On Fig. 8, the flutter speed parameter $v/b\omega_\beta$ is plotted against the inertia parameter. The effect of damping is appreciable and illustrates one method that could be used for eliminating this type of oscillation. On Fig. 9, the frequency ratio $(\omega/\omega_\beta)^2$ is plotted against the inertia parameter. It should be

14 Benjamin Smilg and Lee S. Wasserman, Application of Three-Dimensional Flutter Theory to Aircraft Structures, AC Technical Report, 4798 Air Corps, Materiel Division, Dayton, Ohio.

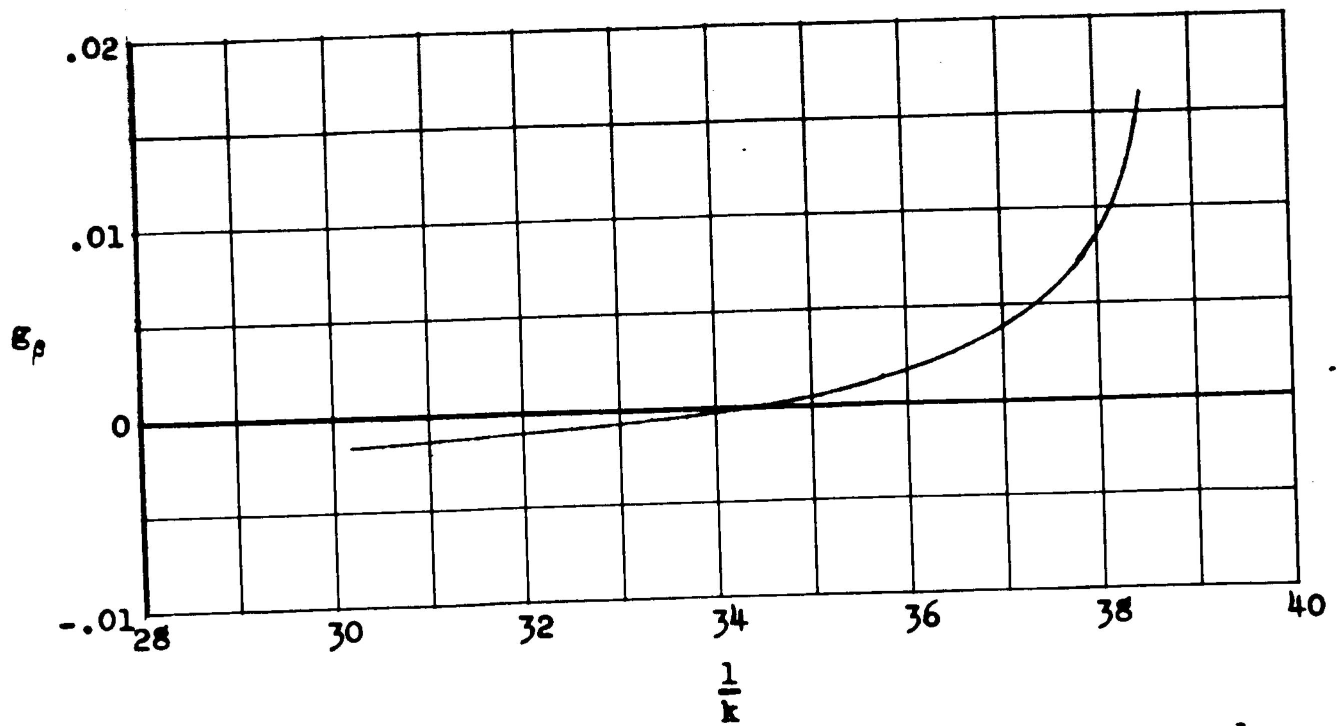


Figure 7.- Plot of damping g_β against reduced frequency $\frac{1}{k}$ illustrating method of determining effect of damping for $c = 0.7$ and $\frac{I_\beta}{\pi\rho b^4} = 20$.

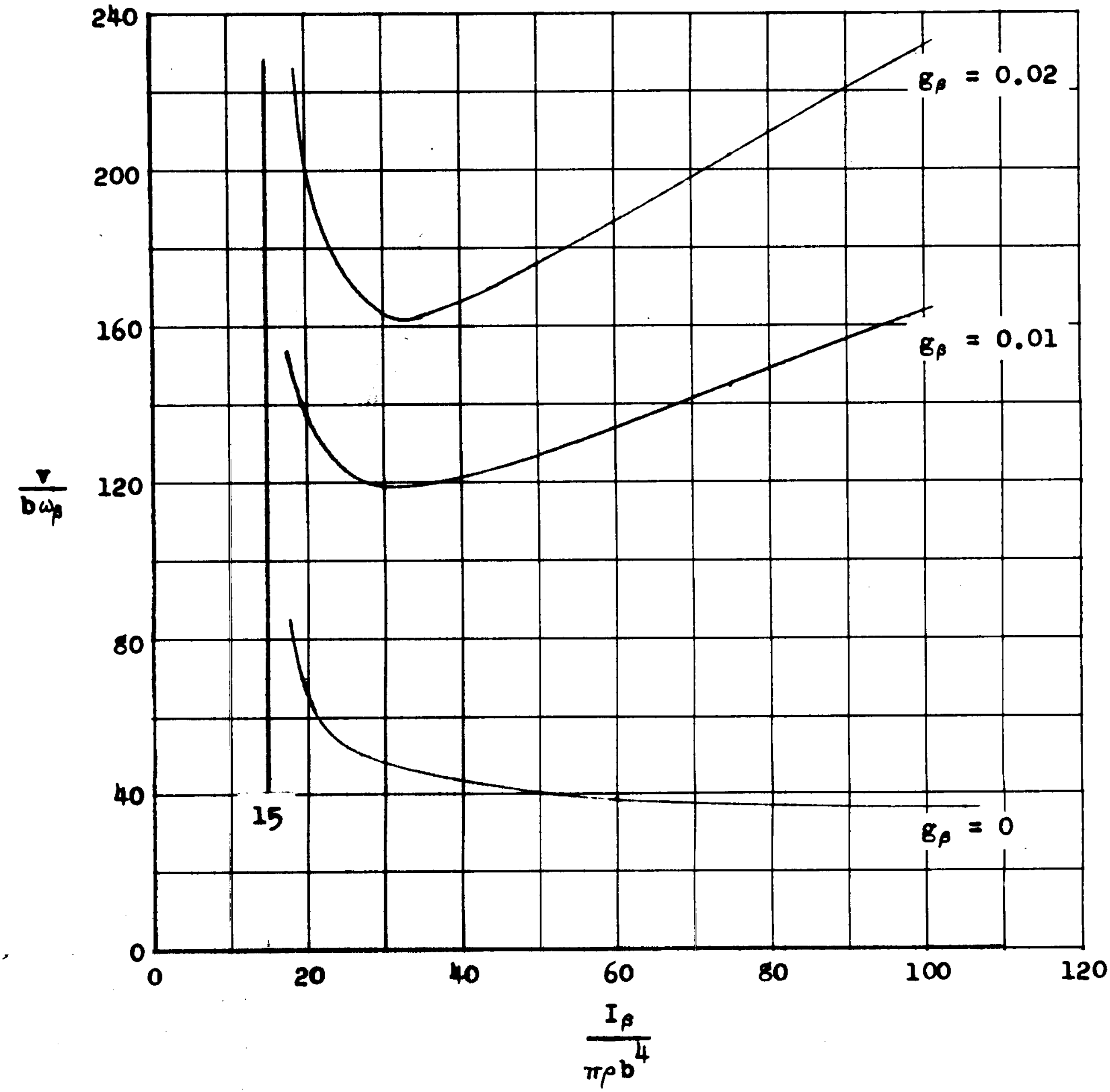


Figure 8.- Plots of flutter speed parameter $\frac{v}{b\omega_\beta}$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for various values of structural damping g_β and $c = 0.7$.

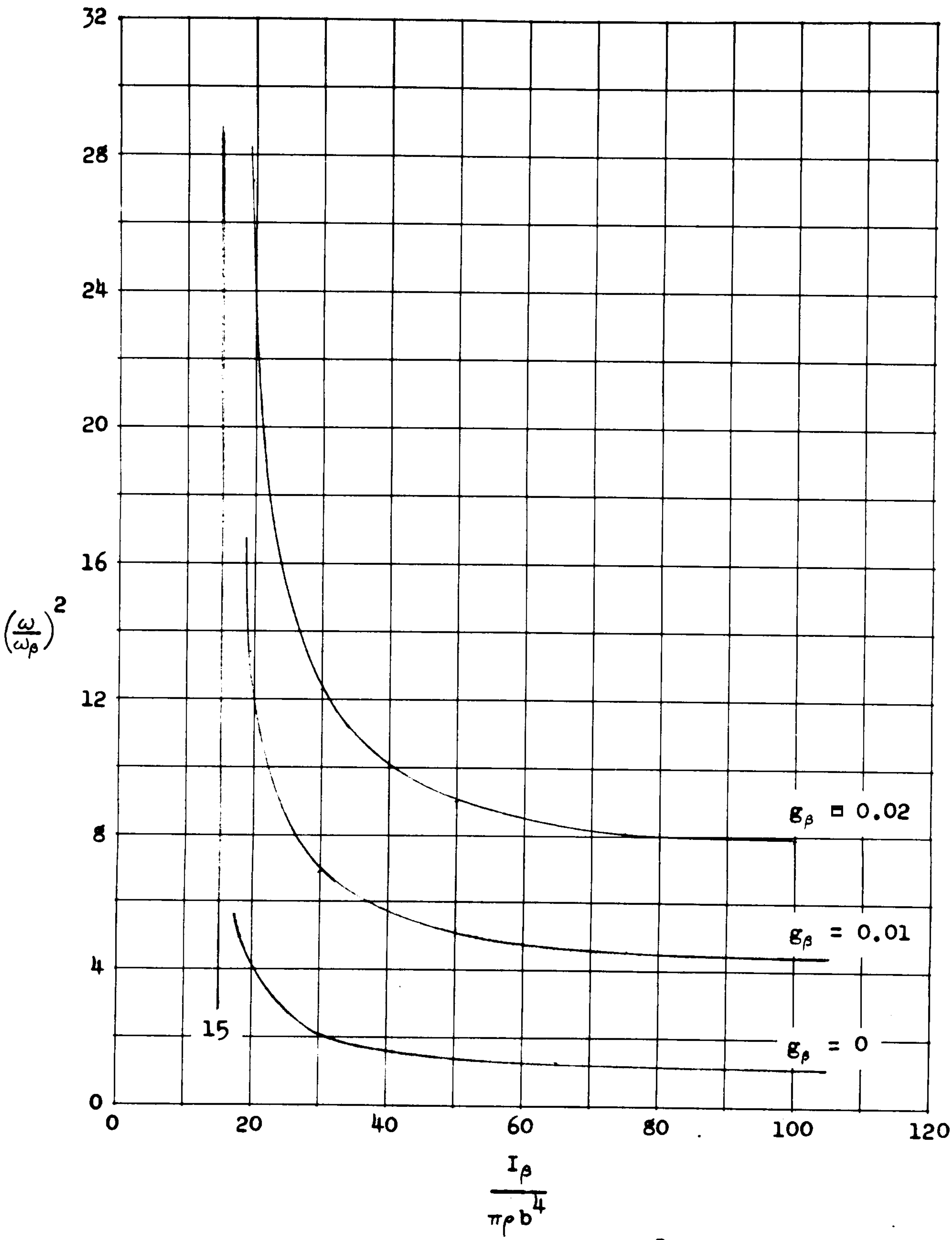


Figure 9 - Plots of frequency ratio $\left(\frac{\omega}{\omega_\beta}\right)^2$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for several values of structural damping g_β and $c = 0.7$.

noted that flutter does not occur at a constant value of the reduced speed $v/b\omega$, as was the situation for the zero damping case. A table of the value of $v/b\omega$ and inertia parameter is shown in Table I for c = .7.

TABLE I

g_β \ $I_\beta/\pi\rho b^4$	20	30	50	75	100
0	34.25	34.25	34.25	34.25	34.25
.01	38.0	44.77	56.1	67.58	77.60
.02	38.80	46.36	58.8	71.50	82.35

Values of Reduced Velocity 1/k for Various Values of Inertia Parameter and Damping

III. EFFECT OF AERODYNAMIC BALANCE

The effect of aerodynamic balance is presented in this section. The term aerodynamic balance usually implies that the control surface is pivoted behind the leading edge, resulting in an overhang. A sketch of a control surface having aerodynamic balance with pertinent dimensions is shown on Fig. 10.

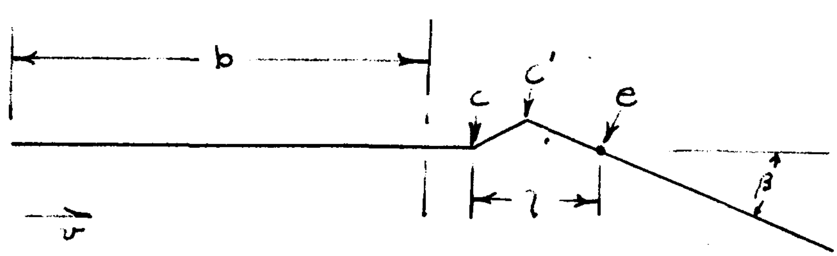


Fig. 10

Control Surface with Aerodynamic Balance

<u>Equilibrium Equation.</u> The moments and forces on a system as shown in Fig. 10 have been derived by Theodorsen and Garrick[15]. The calculation procedure is the same as in the previous sections. The value of 1/k must be determined that satisfies

$$I_{b\beta} = \frac{1}{K}\left\{\frac{1}{\pi^2}\left(T_{19} + \ell T_{27} + \ell^2 T_{29}\right) + \frac{1}{2\pi}\left(T_{12} - 2\ell T_{20}\right) \times \left[\frac{1}{\pi}\left(T_{10} - \ell T_{21}\right)\frac{2G}{K} + \frac{1}{2\pi}\left(T_{11} - 2\ell T_{10}\right) 2F\right]\right\}$$

and the frequency is determined from

$$\left(\frac{\omega}{\omega_\beta}\right)^2 = \frac{1}{1 - R'_{b\beta}\frac{\pi\rho b^4}{I_\beta}}$$

where

$$R'_{b\beta} = -\frac{1}{\pi^2}\left(-T_3 + 2\ell T_2 - \ell^2 T_5\right) + \frac{1}{K^2\pi^2}\left(T_{18} + \ell T_{26} + \ell^2 T_{28}\right) - \frac{1}{2\pi}\left(T_{12} - 2\ell T_{20}\right)\left[\frac{1}{2\pi}\left(T_{11} - 2\ell T_{10}\right)\frac{2G}{K} - \frac{1}{\pi}\left(T_{10} - \ell T_{21}\right)\frac{2F}{K^2}\right]$$

The T coefficients are given in the appendix of the report by Theodorsen and Garrick[16], and all T's are functions of c, except T_{28}. The value of this term is

$$T_{28} = 2\left[1 + c + \log(c, c')\right]$$

where

$$\log\left|\frac{1 - cc' - \sqrt{1 - c'^2}\sqrt{1 + c^2}}{c - c'}\right|$$

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15 Theodore Theodorsen and I. E. Garrick, <u>Nonstationary Flow about a Wing-Aileron-Tab Combination Including Aerodynamic Balance</u>, NACA Technical Report No. 736 (Government Printing Office, Washington, D. C.)

16 Theodore Theodorsen and I. E. Garrick, <u>Loc. Cit.</u>

When $c = c'$ this term is infinite, so that it is not possible to represent the flow over a configuration if c' is directly above c, forming a step. Since the value of c' is usually not known for an actual configuration, some approximation must be made for its determination. Theodorsen and Garrick give an equation for an average aileron as indicated by the following equation

$$c' - c = .25\ (e - c)$$

Results. Calculations have been performed for two axes of rotation e, with both having the same value of c, and the results are plotted in Fig. 11 and Fig. 12. In Fig. 11, the flutter speed coefficient is plotted against $I_\beta / \pi\rho b^4$ for $c = .4$ and $e = .55$ and $e = .75$. The curves are similar to those obtained with no aerodynamic balance, except that the limiting value (vertical asymptote) appears to be at a higher value. In Fig. 12 the frequency ratio $(\omega/\omega_\beta)^2$ is plotted against the inertia parameter for $c = .4$ and $e = .55$ and $e = .75$.

The important facts to be noted are (1) that aerodynamic balance did not eliminate single degree of freedom flutter of the control surface and (2) the greater the amount of aerodynamic balance, the higher the limiting value of the inertia parameter at which the oscillation could begin.

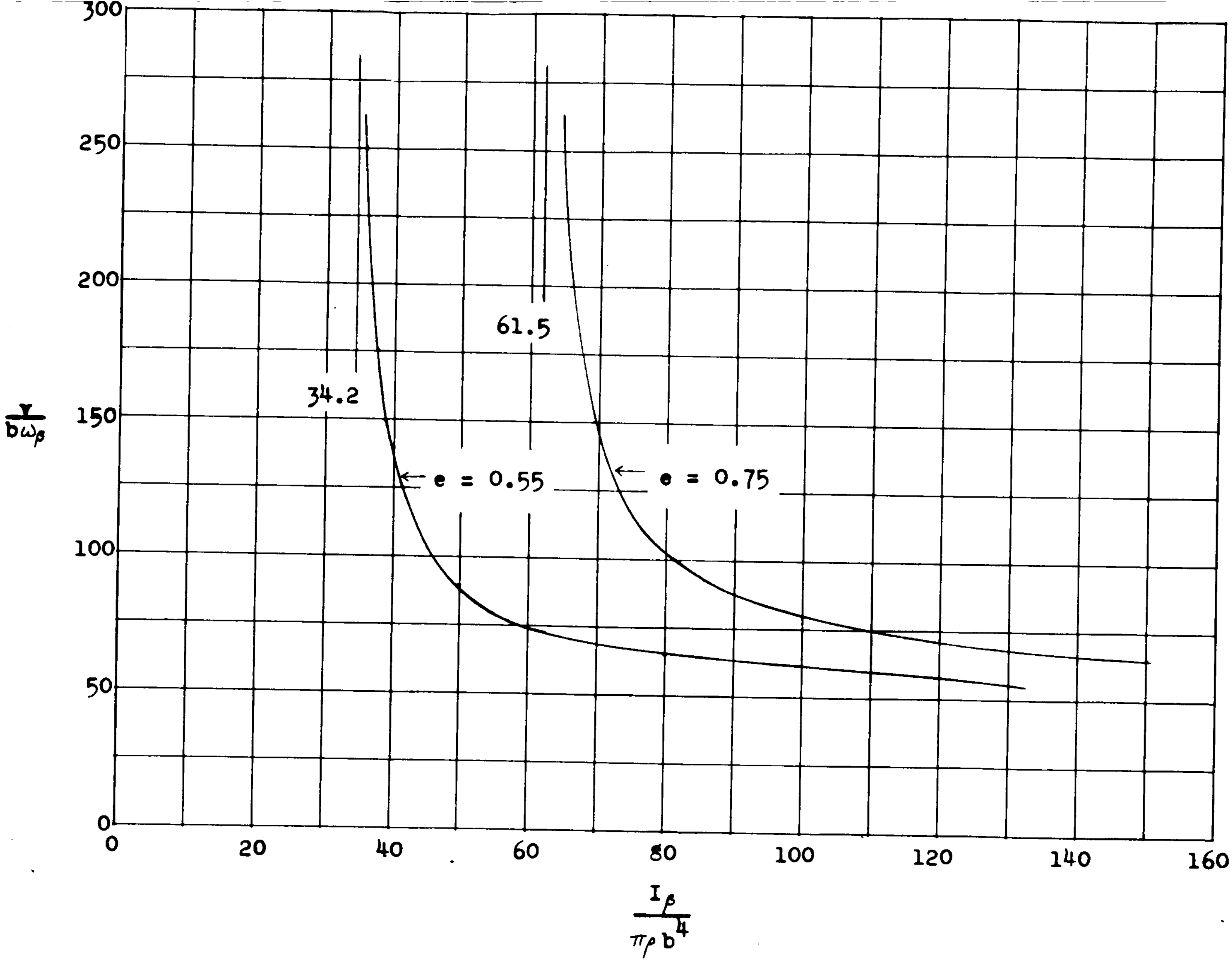

Figure 11.- Plot of flutter speed parameter $\frac{V}{b\omega_\beta}$ against $\frac{I_\beta}{\pi\rho b^4}$ for two different degrees of aerodynamic balance for M = 0 and c = 0.4.

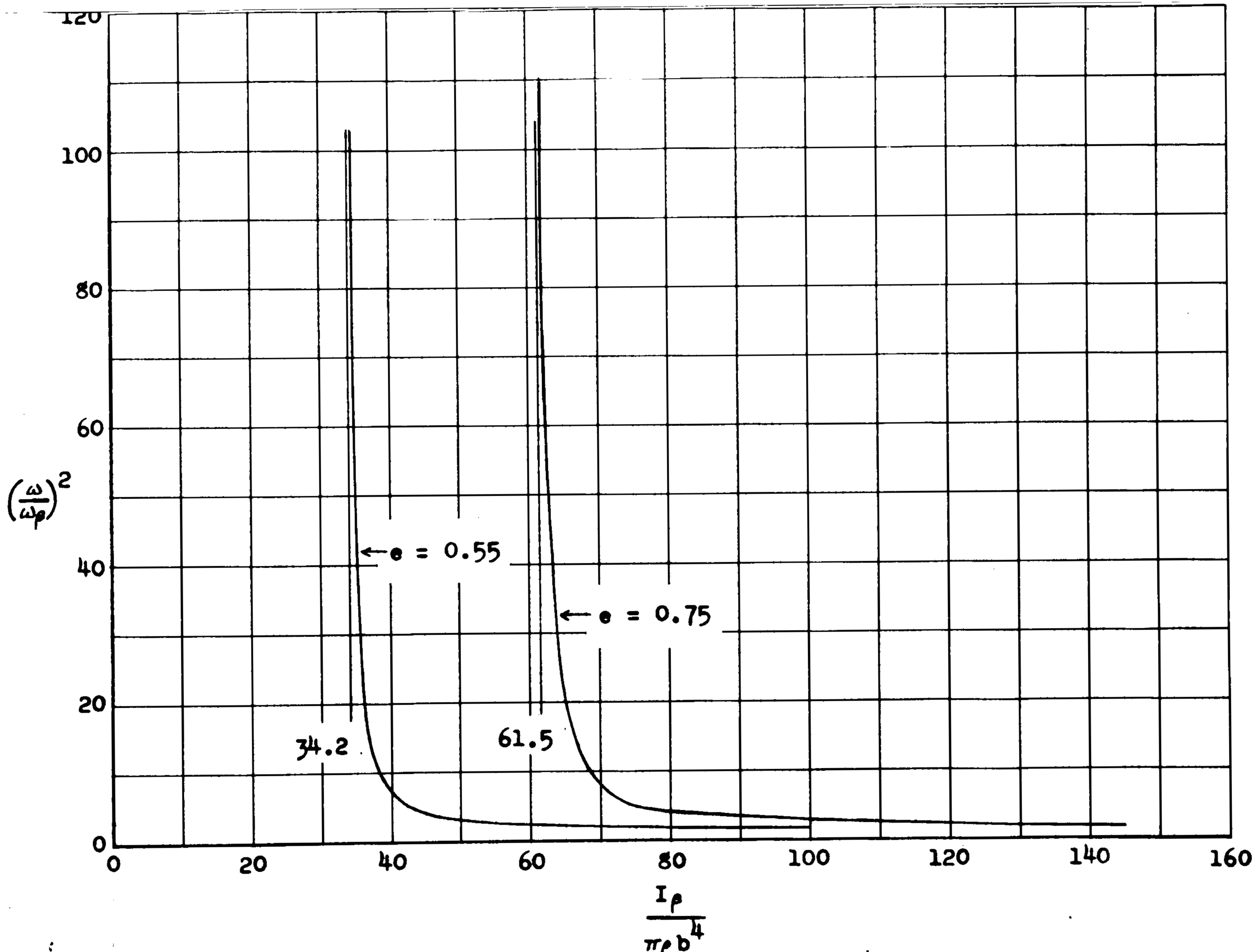

Figure 12.- Plots of $\left(\frac{\omega}{\omega_\rho}\right)^2$ against $\frac{I_\rho}{\pi\rho b^4}$ for two different degrees of

CHAPTER IV

EFFECT OF COMPRESSIBILITY

This chapter deals with the effect of compressibility on single degree of freedom flutter of a control surface for one ratio of control surface chord to wing chord, of 15 percent ($c = .7$). Two Mach numbers are considered, namely $M = .7$ and $M = 10/9$

I. USE OF TABLES FOR CALCULATIONS OF COMPRESSIBILITY EFFECT

Subsonic case $M = .7$. The coefficients for flutter in a compressible medium are given in the form of table and curves in a report by Dietze[17], which appears in a translated form by Air Materiel Command.

A notation differing from that used in this text appears in the Dietze report. The imaginary part of the moment $I_{b\beta}$ (notation of this text) corresponds to the Dietze term K''_{RR} except for a constant. Since the vanishing of the damping moment coefficient, $I_{b\beta}$ represents the borderline condition between damped and undamped oscillations, the vanishing of K''_{RR} will also represent the flutter condition. The real part of the moment $R'_{b\beta}$ (notation of this text) corresponds to the Dietze value K'_{RR} except for a constant multiplier and the relation between the two coefficients is

$$R'_{b\beta} = \frac{K'_{RR}}{k^2} - C_{RR}$$

Therefore the frequency equation is

$$\left(\frac{\omega}{\omega_\beta}\right)^2 = \frac{1}{1 - R'_{b\beta}\frac{\pi\rho b^4}{I_\beta}}$$

---

17 F. Dietze, Die Luftkraefte des Harmonisch Schwingenden Fluegels im Kompressiblem Medium bei Unterschallgeschwindigkeit (Ebenes Problem) Teil II Zahlen-und Kurventafeln (Translated by Headquarters, Air Materiel Command, Wright Field, Dayton, Ohio, No. F-T-S 948-RE

<u>Supersonic case $M = 10/9$.</u> Calculations of the supersonic coefficients for control surface are reported in a paper by Huckel and Durling[18]. The aerodynamic moment on the aileron due to aileron motion is

$$\vec{M}_\beta = -4\rho b^2 v^2 k^2 e^{i\omega t} \beta_0 (N_5 + i N_6)$$

and the summation of moment about the aileron hinge line gives

$$I_\beta \ddot{\beta} + C_\beta \beta = -4\rho b^4 \omega^2 \beta (N_5 + i N_6)$$

The problem again is to find the value of $1/k$ at a given Mach number that would satisfy the equation

$$N_6 = 0$$

In the notation of the supersonic aileron report, X_1, represents the aileron hinge line, measured from the leading edge and non-dimensionalized by the chord. In order to study the same aileron hinge line position as in the subsonic $M = .7$ case, several cross plots are necessary to find the value of $1/k$ that satisfies $N_6 = 0$. For instance, it can be seen that N_6 changes sign for $M = 10/9$, $1/k = .5263$ between aileron hinge position of $X_1 = .9$ and $X_1 = .8$. Similarly for $1/k = .8772$ the damping changes sign between $X_1 = .8$ and $X_1 = .7$. Therefore, a separate plot was made for several values of $1/k$, of N_6 against X_1, and the value of X_1 when $N_6 = 0$ was determined. From these values, a plot of $1/k$ vs X_1, was made and value of $1/k$ at the $X_1 = .85$ was determined.

The frequency may now be determined from the real equation

$$\left(\frac{\omega}{\omega_\beta}\right)^2 = \frac{1}{1 - \frac{4}{\pi} N_5 \frac{\pi\rho b^4}{I_\beta}}$$

---

18 V. Huckel and B. Durling, <u>Tables of Wing-Aileron Coefficients of Oscillating Air Forces for Two-Dimensional Supersonic Flow</u>, NACA Technical Note 2055, Washington, 1950.

where N_5 was determined in the same manner as the value of $1/k$ was found from cross plotting.

II. RESULTS OF CALCULATIONS FOR $M = .7$ and $M = 10/9$

The results of the calculations are presented in Fig. 13, 14, and 15. In Fig. 13, the flutter speed parameter is plotted against the inertia parameter for $M = 0$, $M = .7$ and $M = 10/9$. The curves for $M = .7$ and $M = 10/9$ are similar to the curve for $M = 0$, except that the stable range of the inertia parameter is greatly reduced as the Mach number is increased. Another effect of compressibility is a large reduction in the value of the reduced velocity $1/k$.

In Fig. 14, the frequency ratio is plotted against the inertia parameter for three Mach numbers $M = 0$, $M = .7$ and $M = 10/9$. The major effect of compressibility is to reduce the stable range of the inertia parameter.

A significant plot may be made if the asymptotic value of the inertia parameter is plotted against Mach number as shown in Fig. 15. The relation between the inertia parameter and Mach number is linear, and the region to the right and above the curve is the unstable area. This plot would apply, in particular, to a control surface without elastic restraint ($\omega_\beta = 0$). Therefore, an aileron that is stable at low Mach numbers, could become unstable in the high subsonic or lower supersonic range.

These calculations have been based on potential flow, with no consideration of separated flow phenomena. Accordingly, it is not expected that flutter speeds will be found corresponding exactly to any found

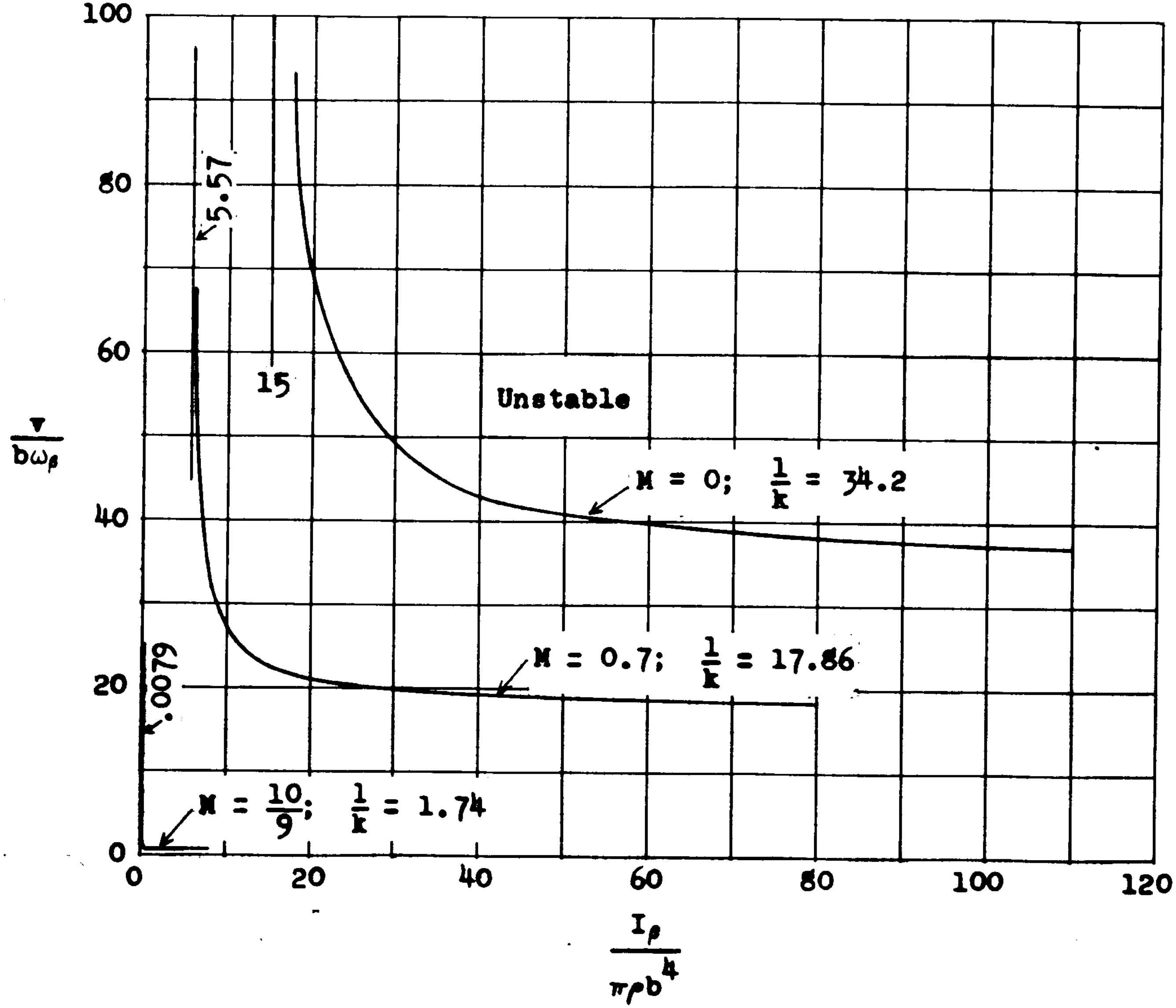

Figure 13.- Plot of flutter speed parameter $\frac{V}{b\omega_\beta}$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for $c = 0.7$ and for several values of Mach number M.

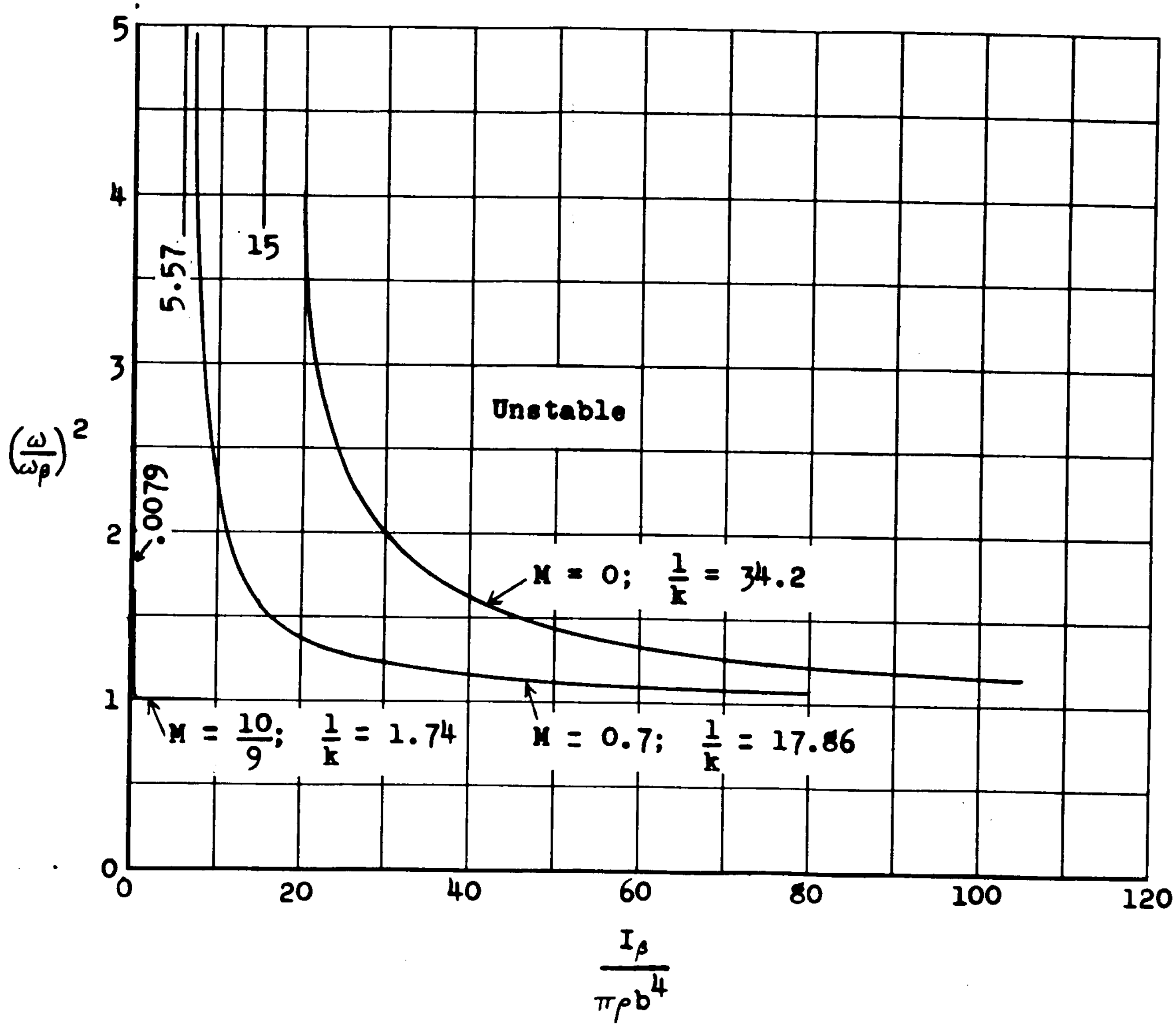

Figure 14.- Plot of frequency ratio $\left(\frac{\omega}{\omega_\beta}\right)^2$ against an inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ for $c = 0.7$ and for several values of Mach number M.

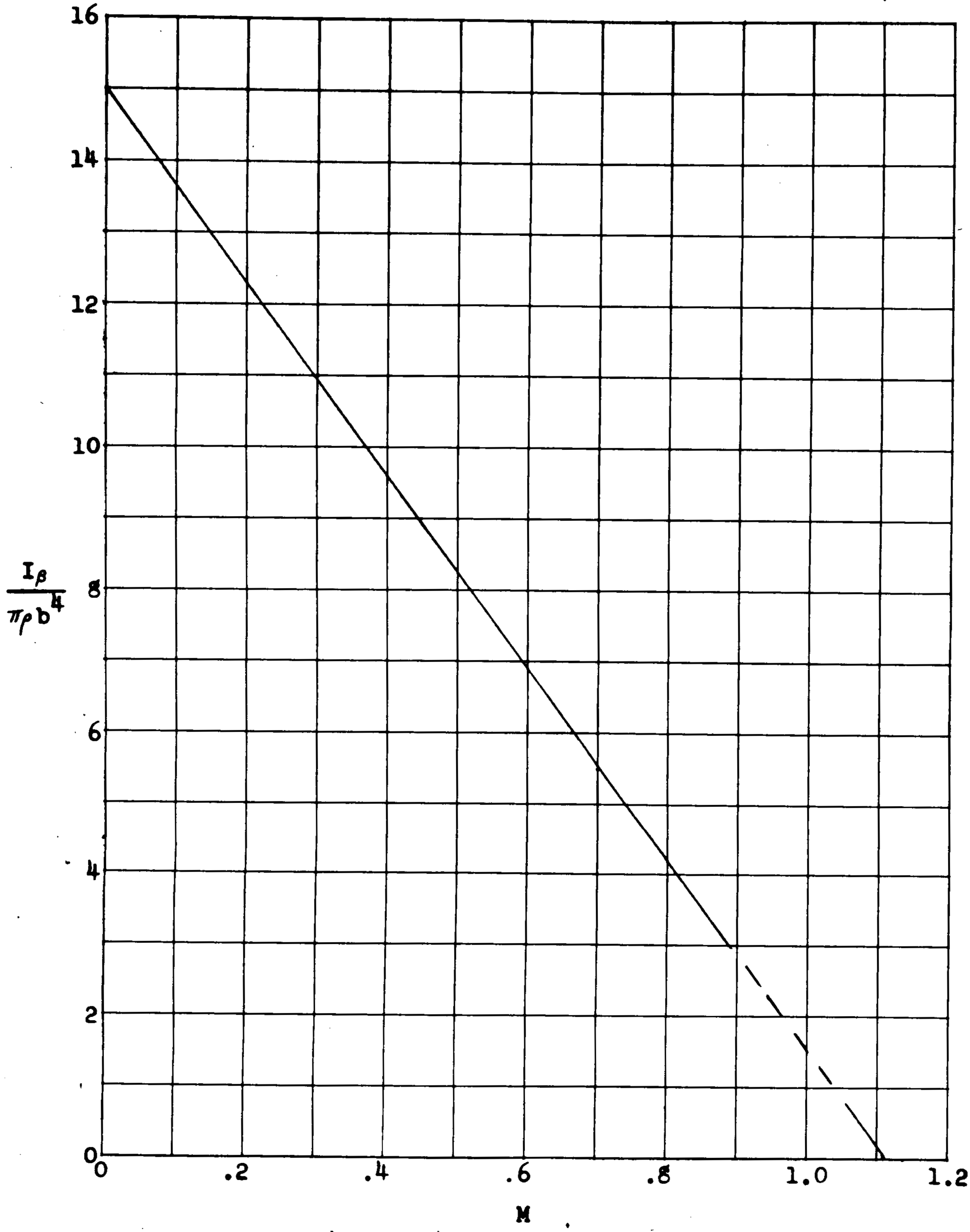

Figure 15.- Asymptotic value of inertia parameter $\frac{I_\beta}{\pi\rho b^4}$ against Mach number for one axis of rotation c=.7

on actual aircraft or determined by experiment. However, the analysis should provide a basis or framework with which experimental data may be more systematically analyzed.

CHAPTER V

SUMMARY AND CONCLUSIONS

I. SUMMARY

The theoretical possibility of single-degree-of-freedom pitching oscillations of a wing based on potential flow has been known (see Section II, Chapter I) for some time. However, corresponding studies of single-degree-of-freedom flutter of a control surface have not been made. The present paper demonstrates by theoretical calculations, that single-degree-of-freedom control surface flutter is possible. The effects of structural damping, aerodynamic balance, axis of rotation, and compressibility are included. Single-degree-of-freedom oscillation of a control surface has been reported on aircraft, and it is thought by some investigators that the cause is in some way connected with separated flow. Even though the calculations of the present report are based on potential flow, the results should provide a logical basis or framework with which to examine separated flow phenomena.

II. CONCLUSIONS

The following conclusions may be enumerated:

1. Single-degree-of-freedom flutter of a control surface is predicted by potential flow theory.
2. Flutter of a control surface of present day aircraft at low Mach numbers and low altitudes would not be likely to occur.
3. Flutter of a control surface is more probable for a configuration operating at high subsonic or low supersonic speeds than at low speeds.

4. This type of oscillatory instability is greatly influenced by an inertia parameter $I_\beta/\pi\rho b^4$, and since this parameter is inversely proportional to fluid density, an increase in altitude could make a control surface that was stable at low altitudes, unstable at high altitudes.
5. Structural damping has a beneficial effect, since it raises the flutter speed appreciably. The use of structural damping may be a convenient method of eliminating flutter.
6. The oscillation is still possible if the control surface is aerodynamically balanced.
7. The results should provide a more logical basis or framework for investigation of oscillations of a control surface thought to be connected with separated flow.

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APPENDIX

SYMBOLS

A_1, A_2, A_3	aerodynamic inertia, damping, and stiffness coefficient for a wing
B_1, B_2, B_3	aerodynamic inertia, damping, and stiffness coefficient for a control surface
a	axis of rotation location of a wing
b	wing half chord, feet
c	control surface hinge axis location, measured from midchord, based on half chord and positive aft, for a control surface without aerodynamic balance
c'	location of leading edge of an aerodynamically balanced control surface, measured from midchord, based on half chord, and positive aft.
C_α	torsional stiffness of wing about axis of rotation
C_β	torsional stiffness of control surface about hinge line c
d	viscous damping coefficient
D	damping coefficient
e	location of hinge axis of an aerodynamically balanced control surface, measured from midchord, based on half chord, and positive aft
F and G	functions of k for oscillating plane flow
g_β	structural damping coefficient ($\pi g_\beta \approx$ logarithmic decrement)
$I_{b\beta}$	out-of-phase aerodynamic moment coefficient

I_α	mass moment of inertia of wing system about axis of rotation, per unit length.
I_β	mass moment of inertia of control surface about hinge per unit length line
K'_{HR} and K''_{HR}	aerodynamic moment coefficients for a control surface in compressible flow
k	reduced frequency
k'	spring constant
l	non-dimensional distance of control surface axis of rotation to leading edge of control surface (e-c)
M_β	total complex aerodynamic moment on control surface
m	mass
M_r	part of inphase aerodynamic moment
N_5, N_6	aerodynamic coefficients for a control surface in supersonic flow
$\overline{R}_{b\beta}$	inphase moment on a control surface
$R'_{b\beta}$	inphase aerodynamic moment on control surface
T_n	control surface coefficients
t	time
v	flutter velocity, ft. per sec.
x	displacement of mass from equilibrium position
α	angular deflection of wing from equilibrium position
β	control surface rotation, measured from wing chord, radians
ρ	fluid density, slugs per cu. ft.

ω circular flutter frequency, radians per sec.

ω_β natural circular frequency of control surface, about hinge axis, radians per sec.